博士学位论文

复杂偏好下多属性大群体决策方法研究

Large Group Decision-making Method with Complex Preference

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2013 6

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摘要：

:

(1)

J- E- C-

(2)

2002-2009

0-1

(3)

15

40 181

关键词：

分类号： C934;TP311.52

Large Group Decision-making Method with Complex Preference

Abstract: In recent year, the natural disaster and public crisis continue to occur, which caused signi¯cant damage to the national economy and people's lives and property, so emergency decision making problem become the focus and emphasis of decision science. We analyze the di®erence between the emergency decision-making problem and other decision-making problem, according to the emergency decision environment's complexity and variety, the decision task's complexity, variety, uncertainty and timeliness, and the complex relationship between the decision attribute, we give the de¯nition of complex preference. Using the similarity measure to solve the multiple attribute group decision making problem with complex preference, we also compare our method with the other group decision making method, the numerical results show our method's merits. Thesis work mainly includes the following aspects:

(1) Deterministic preferences. We mainly study multiple attribute large group decision making (MALGDM) problem with vector preference. Firstly, we introduce the J-similarity measure, E-similarity measure and C-similarity measure, there are some defects in the above three similarity measures, so we provide the variational coe±cient similarity measure based on the three similarity measure, which satisfy re°exivity, symmetry and boundedness. Then, we apply the variation coe±cient similarity to evaluate the emergency management capability of major snow disaster in Hunan province of China. For comparison, the decision results of di®erent similarity measures are also given, which show that our method have better similarity identi¯cation and the proposed method is applicable.

(2) Associated preference. Consider the association between the

decision attribute, we discus the hierarchical relationship between the decision attribute. A new method based on the trigonometric functions to determine the weight of decision attribute is proposed. By using the method of preference aggregation for large group decision making, we get a new model for evaluating grade of natural disaster loss based on the hierarchy of evaluation index and its algorithm. Lastly, the paper presented the result of dividing gradation of the condition of the natural disaster for 2002-2009 in Chinese. We introduce a new method of large group decision based on binary relationship attributes. By using binary relation of the decision attributes, the relation matrix which is a 0-1 matrix is formed. With the property of 0-1 matrix and the norm of the matrix, this paper establishes a new large group preference aggregating model and proposes the method of evaluation schemes ranking. Finally, an example of Hunan province emergency management capability of major snow disaster is presented to authenticate the method of MALGDM.

(3) Fuzzy preference. We give a detailed introduction about MALGDM problem with intuitionistic fuzzy set preference, triangular fuzzy preference, intuitionistic triangular fuzzy preference, trapezoidal fuzzy linguistic preference, intuitionistic trapezoidal fuzzy preference, intuitionistic trapezoidal fuzzy binary preference, internal trapezoidal fuzzy preference. We provide the method to compute the weight of the attribute with fuzzy information. By constructing the corresponding weighted similarity model, we get the order of each alternative and the best alternative. Finally, we apply the above methods to solve the emergency decision-making problem, and compare with the other decision-making methods, the decision results show our method is e®ective and applicable. There are 15 ¯gures, 40 tables and 181 references.

Keywords: Multiple attribute decision-making, Emergency decision-making, Complex preference, Larger group, Similarity measure

Classification: C934;TP311.52

目　录

符号说明

A

B

C

C^k k

D

E

F

G

H

i i

j j

k k

m

n

O

p

R

S

T

U

V

W

X Y

Z 0-1

1 导论

1.1 选题背景及研究意义

71171202

70921001

70871121

12AZD109

13.5 10973.9

2009 4.8 2523.7 2010 4.3

5339.9 2011 4.4 3110.3 2012

2012

11336.1 773.8

1.2 国内外研究现状综述

[1]

1.2.1 复杂偏好下多属性大群体决策问题的研究现状

[2], [3]-[8]

[9]

Zahir [10]

Basak Saaty [11]

Xu [12] AWA

[13] AHP [14]

[15]

[22] 6
6

Wang [23]

M(A, B)[22] L(A, B)[22] K(A, B)[24] C(A, B)[24] Wang

Atanassow 1986
[25]

Hamming Hausdor® Euclidean

Cosine
Li Cheng [26]
Liang Shi 2003
[27]

Szmidt Kacprzyk
[28] Huang Yang Hausdor®
Hausdor® [29]
Xu

[30] Ye [31]

[32]

Shu
[33] Abbasbandy Hajjari [34]

[35]
[36]

[37]
Liu [38]

Nehi Maleki 2005
[39] Ye
[40]

Ye [41]

[42] Bellman Zadeh 1970
[43]

[44] [45] TOPSIS [46] [47] [48]
[49] 2008

1.4 论文的总体结构

5.2

J- E- C-

J-

E- C-

E- 5.3 [31] 13

5.4 E-

5.5

5.6

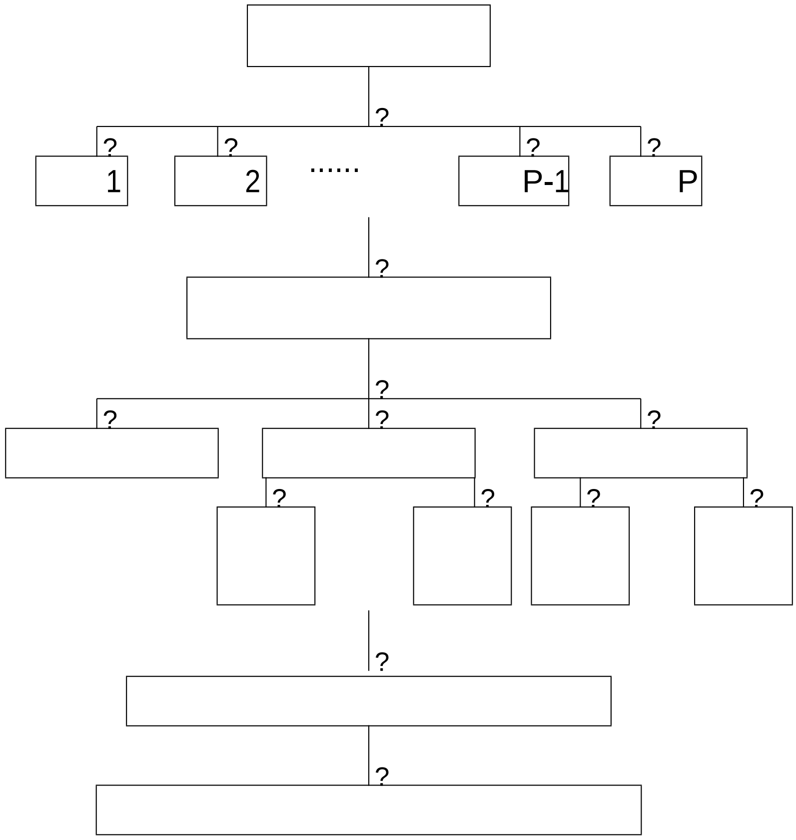

图 1-1 论文的研究框架

[50] [51] [52]

[53] [54] [55] [41]

[38]

[56]–[58] [41] [38]

7.1

7.2

7.3

7.4

7.5

1-1

2 基于矢量偏好的多属性大群体决策方法

2.1 引言

[2]

Normal Group Technique, NGT

Delphi AHP [3]

[59] [8]

Xu [12] AWA

[13] AHP

[14]

[15]

2.2 变系数相似度模型

$X = (x_1, x_2, \ldots, x_n)$ $Y = (y_1, y_2, \ldots, y_n)$ R n

Jaccard 1901 J- [60]

定义2.1 X Y J

$$J(X,Y) = \frac{X Y}{\|X\|_2^2 + \|Y\|_2^2 - X Y} = \frac{\sum\limits_{i=1} x_i y_i}{\sum\limits_{i=1} x_i^2 + \sum\limits_{i=1} y_i^2 - \sum\limits_{i=1} x_i y_i}, \qquad (2\text{-}1)$$

$X Y = \sum\limits_{i=1} x_i y_i$ X Y , $\|X\|_2 = \sqrt{\sum\limits_{i=1} x_i^2}$ $\|Y\|_2 = \sqrt{\sum\limits_{i=1} y_i^2}$ X Y Euclidean (L_2).

Dice E- [61] 2.2

定义2.2 E- :

$$E(X;Y) = \frac{2XY}{\|X\|_2^2 + \|Y\|_2^2} = \frac{2\sum_{i=1} x_i y_i}{\sum_{i=1} x_i^2 + \sum_{i=1} y_i^2}. \quad (2\text{-}2)$$

Salton McGill C- [62] 2.3

定义2.3 C- :

$$C(X;Y) = \frac{XY}{\|X\|_2\|Y\|_2} = \frac{\sum_{i=1} x_i y_i}{\sqrt{\sum_{i=1} x_i^2}\sqrt{\sum_{i=1} y_i^2}}. \quad (2\text{-}3)$$

2.1 2.2 $x_i = y_i = 0$ i $(i = 1, 2, \ldots, n)$ $x_i = 0$ $y_i = 0$ $(i = 1, 2, \ldots, n)$ 2.3 Consine 2.4

定义2.4 $X = (x_1, x_2, \ldots, x_n)$ $Y = (y_1, y_2, \ldots, y_n)$ R n

$$\begin{aligned} S(X;Y) &= \alpha\frac{2XY}{\|X\|_2^2 + \|Y\|_2^2} + (1-\alpha)\frac{XY}{\|X\|_2\|Y\|_2} \\ &= \alpha\frac{2\sum_{i=1} x_i y_i}{\sum_{i=1} x_i^2 + \sum_{i=1} y_i^2} + (1-\alpha)\frac{\sum_{i=1} x_i y_i}{\sqrt{\sum_{i=1} x_i^2}\sqrt{\sum_{i=1} y_i^2}}, \end{aligned} \quad (2\text{-}4)$$

$\alpha = 0$ E- $\alpha = 1$ C- 2.4

2.4 X Y 1 2 3 2.1.

定理 2.1. 设$X = (x_1, x_2, \ldots, x_n)$和$Y = (y_1, y_2, \ldots, y_n)$是实数集$R$上的$n$维矢量，两矢量间的相似度满足下面的性质：

*(P1)***自反性：若**$X = Y$，**即：**$x_i = y_i$，**对任意的**$i = 1, 2, \ldots, n$，**那么**$S(X;Y) = 1$；

*(P2)***对称性：**$S(X;Y) = S(Y;X)$；

*(P3)***有界性：**$0 \le S(X;Y) \le 1$.

证明. (P1) $X=Y$, $x_i=y_i$, $i=1,2,\dots,n$

$$S(X,X)=\alpha\frac{2XX}{\|X\|_2^2+\|X\|_2^2}+(1-\alpha)\frac{XX}{\|X\|_2\|X\|_2}$$
$$=\alpha\frac{2\sum_{i=1}^{n}x_ix_i}{\sum_{i=1}^{n}x_i^2+\sum_{i=1}^{n}x_i^2}+(1-\alpha)\frac{\sum_{i=1}^{n}x_ix_i}{\sqrt{\sum_{i=1}^{n}x_i^2}\sqrt{\sum_{i=1}^{n}x_i^2}}$$
$$=\alpha+1-\alpha=1. \tag{2-5}$$

(P2)

$$S(X,Y)=\alpha\frac{2XY}{\|X\|_2^2+\|Y\|_2^2}+(1-\alpha)\frac{XY}{\|X\|_2\|Y\|_2}=S(Y,X).$$

(P3) $S(X,Y)\geq 0$ $S(X,Y)\leq 1$.

$$0\leq\frac{2XY}{\|X\|_2^2+\|Y\|_2^2}\leq 1, 0\leq\frac{XY}{\|X\|_2\|Y\|_2}\leq 1, \tag{2-6}$$

(2.6) (2.4)

$$S(X,Y)\leq\alpha+(1-\alpha)=1. \tag{2-7}$$

□

2.3 几种不同相似度之间的比较

2.2

18 2-1 V_p=(0.8542, 0.3460, 0.7000, 0.789, 0.1234, 0.6534)

α=0.1 α=0.5 α=0.75 $\alpha=0.9$ 2.1 - 2.4

18 V^i ($i=1,2,\dots,18$) V_p J- E- C-

S- Matlab

2-1 - 2-4

2-1 2-3 α($\alpha=0.1$, $\alpha=0.5$, $\alpha=0.75$)

Jcaard (J-similarity) Dice (E-similarity)

(S-similarity) 2-4 α=0.9 Jcaard

(J-similarity) Dice (E-similarity) Consine C-similarity)

(S-similarity)

C-

2.4 C-

表 2-1 6个群体成员的偏好矢量

	1	2	3	4	5	6
V^1	0.8452	0.9037	1.0000	1.0000	0.8003	0.9316
V^2	0.4410	0.6640	0.4722	0.6227	0.5933	0.6010
V^3	0.8613	0.7228	0.7569	0.3666	0.7118	0.9731
V^4	1.0000	0.0000	0.3151	0.1609	0.5933	0.0432
V^5	0.6810	0.9374	0.6152	0.8043	0.7031	1.0000
V^6	0.1113	1.0000	0.0000	0.1963	0.3100	0.5725
V^7	0.6206	0.7769	0.4711	0.5180	0.3172	0.4194
V^8	0.9055	0.9256	0.6170	0.8522	1.0000	0.7818
V^9	1.0000	0.8582	0.7363	0.8865	0.5421	0.8721
V^{10}	0.9786	0.7769	0.8008	0.7156	0.5039	0.4552
V^{11}	0.3351	0.2397	0.2104	0.2844	0.2282	0.2720
V^{12}	0.1273	0.6164	0.4122	0.3690	0.1098	0.4389
V^{13}	0.0000	0.2996	0.3252	0.4216	0.1683	0.2370
V^{14}	0.4397	0.4730	0.3241	0.3034	0.4352	0.6344
V^{15}	0.5536	0.6292	0.6152	0.6836	0.7534	0.8070
V^{16}	0.3338	0.0813	0.4122	0.1254	0.1098	0.6344
V^{17}	0.8311	0.8716	0.4711	0.6387	0.5933	0.6360
V^{18}	0.3847	0.8716	0.3316	0.3034	0.4352	0.6344

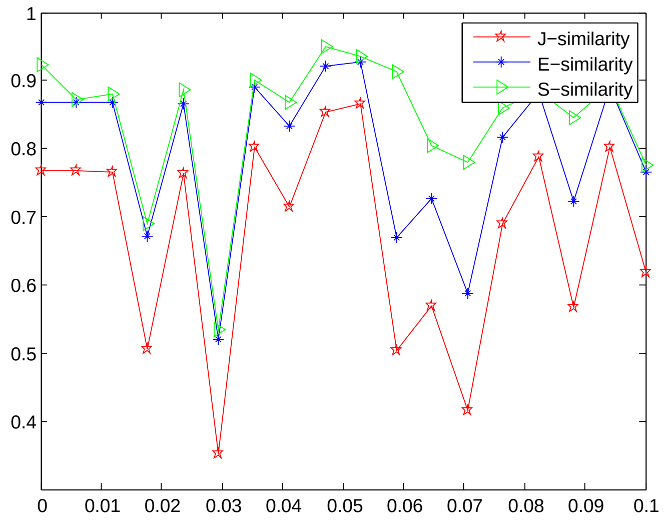

图 2-1 α=0.1时的三种相似度曲线: J-相似度, E-相似度, S-相似度

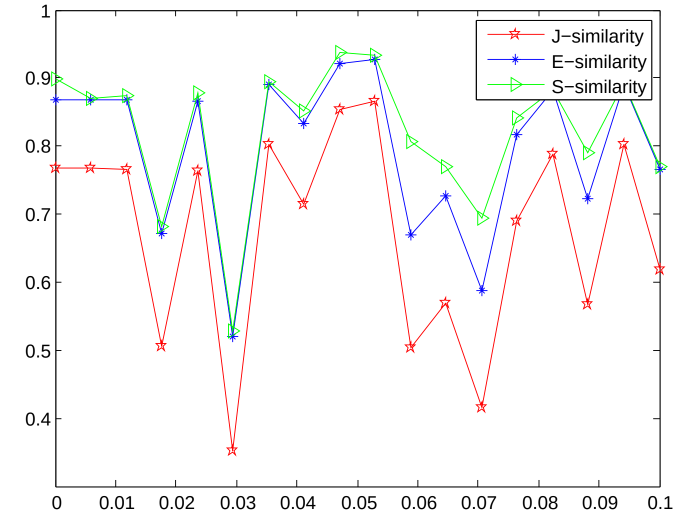

图 2-2 α=0.5时的三种相似度曲线: J-相似度, E-相似度, S-相似度

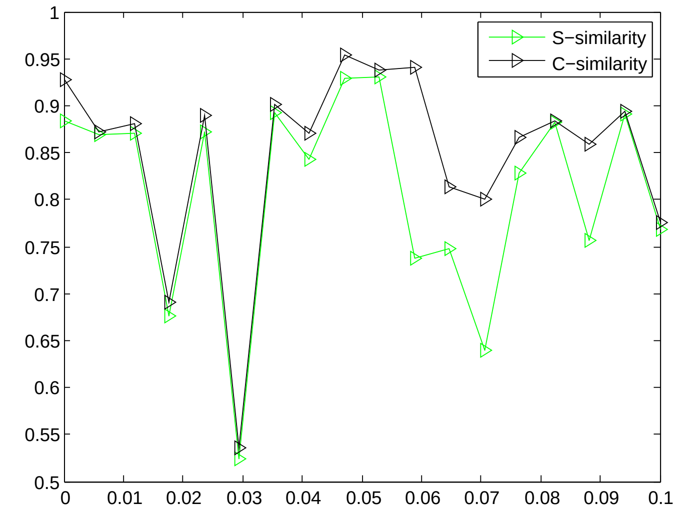

图 2-3 α=0.75时的S-相似度和C-相似度曲线

2.4 变系数相似度在应急决策中的应用

2.4.1 决策问题描述

p $A = \{A_1, A_2, \ldots, A_p\}$ n
$C = \{C_1, C_2, \ldots, C_n\}$ m - p
n C_j k i v_{ij}^k
$v_{ij}^k \geq 0$ $i = 1, 2, \ldots, p; j = 1, 2, \ldots, n; k = 1, 2, \ldots m$ $V_i^k = (v_{i1}^k, v_{i2}^k, \ldots v_{in}^k)$
- k A_i

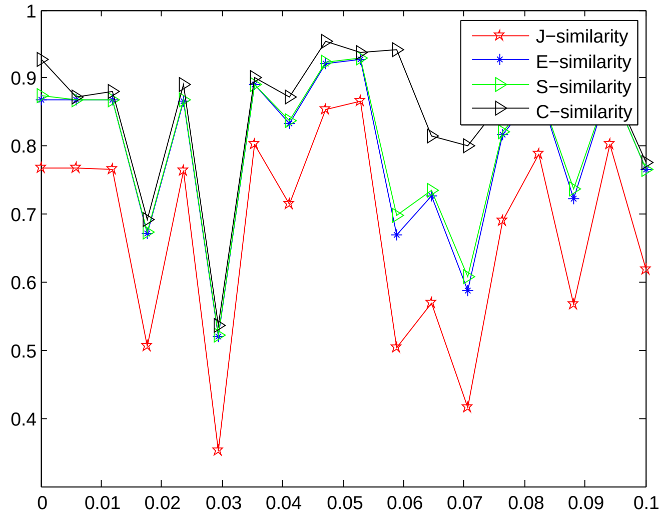

图 2-4 α=0.9时的四种相似度曲线

2.4.2 应急决策案例分析

[63]

3 6

6

6 18 6 1 1

(2) 2 3 3

4 4 5 5

6 6

步骤1 2-8 [63] 2-1

$$Z_{ij} = \frac{y_j^{max} - y_{ij}}{y_j^{max} - y_j^{min}}, \qquad (2\text{-}8)$$

$(V^1 - V^6)$

$(V^7 - V^{12})$ $(V^{13} - V^{18})$ 2-1

步骤2 V_p=(0.8542, 0.346, 0.7, 0.789, 0.1234, 0.6534) $\alpha = 0.75$ 2-1 - 2-4 18 V^i $(i = 1, 2, ..., 18)$ 2-2

步骤3 2-2 $V^i (i = 1, 2, ..., 18)$

表 2-2 18个偏好矢量与理想方案偏好矢量之间的四种相似度

	J-	E-	C-	S-
1	0.7671	0.8682	0.9283	0.8832
2	0.7671	0.8682	0.8728	0.8694
3	0.7661	0.8675	0.8812	0.8710
4	0.5051	0.6712	0.6911	0.6762
5	0.7628	0.8654	0.8898	0.8715
6	0.3512	0.5199	0.5355	0.5238
7	0.8015	0.8898	0.9008	0.8926
8	0.7140	0.8332	0.8713	0.8427
9	0.8530	0.9207	0.9542	0.9290
10	0.8660	0.9282	0.9372	0.9304
11	0.5040	0.6703	0.9411	0.7380
12	0.5700	0.7261	0.8141	0.7481
13	0.4152	0.5867	0.8003	0.6401
14	0.6898	0.8165	0.8655	0.8287
15	0.7892	0.8822	0.8843	0.8827
16	0.5663	0.7231	0.8591	0.7571
17	0.8029	0.8907	0.8937	0.8915
18	0.6191	0.7648	0.7762	0.7676

$T_1 = (t_{ij})_{6\times4}$:

$$T_1 = \begin{pmatrix} 0.7671 & 0.8682 & 0.9283 & 0.8832 \\ 0.7671 & 0.8682 & 0.8728 & 0.8694 \\ 0.7661 & 0.8675 & 0.8812 & 0.8710 \\ 0.5051 & 0.6712 & 0.6911 & 0.6762 \\ 0.7628 & 0.8654 & 0.8898 & 0.8715 \\ 0.3512 & 0.5199 & 0.5355 & 0.5238 \end{pmatrix};$$

$T_2 = (t_{ij})_{6\times4}$:

$$T_2 = \begin{pmatrix} 0.8015 & 0.8898 & 0.9008 & 0.8926 \\ 0.7140 & 0.8332 & 0.8713 & 0.8427 \\ 0.8530 & 0.9207 & 0.9542 & 0.9290 \\ 0.8660 & 0.9282 & 0.9372 & 0.9304 \\ 0.5040 & 0.6703 & 0.9411 & 0.7380 \\ 0.5700 & 0.7261 & 0.8141 & 0.7481 \end{pmatrix};$$

$T_3 = (t_{ij})_{6\times4}$:

$$T_3 = \begin{pmatrix} 0.4152 & 0.5867 & 0.8003 & 0.6401 \\ 0.6898 & 0.8165 & 0.8655 & 0.8287 \\ 0.7892 & 0.8822 & 0.8843 & 0.8827 \\ 0.5663 & 0.7231 & 0.8591 & 0.7571 \\ 0.8029 & 0.8907 & 0.8937 & 0.8915 \\ 0.6191 & 0.7648 & 0.7762 & 0.7676 \end{pmatrix};$$

6 [63] $W = (0.162, 0.198, 0.201, 0.157, 0.174, 0.108)$

$$WS = WT. \tag{2-9}$$

步骤4 2-9 2-3

表 2-3 α=0.75时，四种相似度的决策结果

	WJ-	WE-	WC-	WS-
	0.6801	0.7990	0.8215	0.8046
	0.7279	0.8349	0.9091	0.8535
	0.6579	0.7851	0.8530	0.8021
	$Z > C > X$	$Z > C > X$	$Z > X > C$	$Z > C > X$

2-3

J- E-

C-

$\alpha = 0.25$ 2-9

2-4

表 2-4 $\alpha = 0.25$时，四种相似度的决策结果

	WJ-	WE-	WC-	WS-
	0.6801	0.7990	0.8215	0.8360
	0.7279	0.8349	0.9091	0.8905
	0.6579	0.7851	0.8530	0.8159
	$Z > C > X$	$Z > C > X$	$Z > X > C$	$Z > C > X$

2-3 2-4 J- E-

C-

2- C-

2.5 本章小结

J- E- C-

2.4

2-3 2-4

J- E-

C-

3 基于属性层次关系偏好的多属性大群体决策方法

3.1 决策问题研究背景

[64]

[65] [66]

[67] [68]

4

[69]

[70] [71]

3.2 基于属性层次关系的属性权重确定方法

n i n_i n_i

$C_i^1; C_i^2; ...; C_i^{n_i}$ i C_i^j

$$w_i^j = \sin(\frac{j}{n_i}¼) + \cos(\frac{j}{n_i}¼) + p\overline{2}; \quad (3\text{-}1)$$

C_i^j :

$$\tilde{w}_i^j = \frac{w_i^j}{\sum_{j=1}^{n_i} w_i^j}. \tag{3-2}$$

定理 3.1. 证明公式 (*3-1*) 和 (*3-2*) 式满足权重的定义，并且由w_i^j 定义的函数是减函数，满足权重重要性递减，其值也递减。

证明. $\sum_{j=1}^{n_i} w_i^j = 1$ $w_i^j \geq 0$ w_i^j

$$\begin{aligned} w_i^j &= \sin(\frac{j}{n_i}\pi) + \cos(\frac{j}{n_i}\pi) + \sqrt{2} \\ &= \sqrt{2}[\frac{\sqrt{2}}{2}\sin(\frac{j}{n_i}\pi) + \frac{\sqrt{2}}{2}\cos(\frac{j}{n_i}\pi)] + \sqrt{2} \\ &= \sqrt{2}\cos(\frac{\pi}{4} + \frac{j}{n_i}\pi) + \sqrt{2}. \end{aligned} \tag{3-3}$$

$$-1 \leq \cos(\frac{\pi}{4} + \frac{j}{n_i}\pi) \leq 1; \tag{3-4}$$

3-4 3-1 $w_i^j \geq 0$ $\cos(\frac{\pi}{4} + \frac{j}{n_i}\pi)$ $[0, \pi]$ j 3-1

□

3.3 基于属性层次关系的偏好矢量集结

m

图 3-1 自然灾害等级评价体系

定义3.1 . a n v^j $(j = 1, 2, \ldots, n)$ $V^j = (v^1, v^2, \ldots, v^n)$

a p $T' = [V^1, V^2, \ldots, V^p]$

m i n_i

m m n_m $m-1$ n_{m-1}

$m-1$

$$T_{m-1}^1 = \begin{pmatrix} t_{11} & t_{12} & \ldots & t_{1n_{m-1}} \\ t_{21} & t_{22} & \ldots & t_{2n_{m-1}} \\ \vdots & \vdots & \vdots & \vdots \\ t_{n_m1} & t_{n_m2} & \ldots & t_{n_mn_{m-1}} \end{pmatrix};$$

$m-2$

$$\begin{aligned} V_{m-2}^1 = W_m T_m^1 &= (w_m^1, w_m^2, \ldots, w_m^{n_m}) \begin{pmatrix} t_{11} & t_{12} & \ldots & t_{1n_{m-1}} \\ t_{21} & t_{22} & \ldots & t_{2n_{m-1}} \\ \vdots & \vdots & \vdots & \vdots \\ t_{n_m1} & t_{n_m2} & \ldots & t_{n_mn_{m-1}} \end{pmatrix} \\ &= (v_{m-1}^{11}, v_{m-1}^{12}, \ldots, v_{m-1}^{1n_{m-1}}). \end{aligned} \tag{3-5}$$

3-5 $m-2$ n_{m-2}

$m-2$

3.4 基于属性层次关系的大群体应急决策方法

3.4.1 基于属性层次关系的评价模型的算法

l $a^i\ (i = 1, 2, \ldots, l)$ m

步骤1 l

$$Z_{ij} = \frac{v^{ij} - v^{1j}}{Max(v^j) - v^{1j}}(1.00 - {}^-) + {}^- \tag{3-6}$$

$i = 1, 2, \ldots, l; j = 1, 2, \ldots, l$

步骤3 $i = 1$ $k = m$ m $m - 1$ n_{m-1} T_k^r $(i = 1, 2, \ldots, n_{m-1})$ 3-5 $m - 2$ n_{m-2} $k = m - 1$ $k = 0$ $m - 2, m - 3, \ldots, 2$

步骤4 3 3-5

$$T_2 = \begin{pmatrix} t_{11} & t_{12} & \ldots & t_{1n_1} \\ t_{21} & t_{22} & \ldots & t_{2n_1} \\ \vdots & \vdots & \vdots & \vdots \\ t_{n_2 1} & t_{n_2 2} & \ldots & t_{n_2 n_1} \end{pmatrix};$$

$$V_1 = W_2 T_2 = (w_2^1, w_2^2, \ldots, w_2^{n_2}) \begin{pmatrix} t_{11} & t_{12} & \ldots & t_{1n_1} \\ t_{21} & t_{22} & \ldots & t_{2n_1} \\ \vdots & \vdots & \vdots & \vdots \\ t_{n_2 1} & t_{n_2 2} & \ldots & t_{n_2 n_1} \end{pmatrix}$$
$$= (v_1^1, v_1^2, \ldots, v_1^{n_1}).$$

a_1

$$o_1 = W_1 (V_1)' = (w_1^1, w_1^2, \ldots, w_1^{n_1}) \begin{pmatrix} v_1^1 \\ v_1^2 \\ \ldots \\ v_1^{n_1} \end{pmatrix} \tag{3-7}$$

步骤5 $i = i + 1$ 3- 4 $i > l$ l $o_1, o_2, \ldots, o_l$ $O = (o_1, o_2, \ldots, o_l)$ $0 \leq o_i \leq 1$ $o_i > 1$ O

步骤6 $\Delta = \{\Delta_1, \Delta_2, \ldots, \Delta_r\}$ $\frac{r-1}{r} \leq o_i \leq \frac{r-2}{r}$ Δ_2 3-1.

表 3-1 评价等级划分

	1	2	...	i	$i+1$	...	r
	$(1, \frac{r-1}{r})$	$(\frac{r-1}{r}, \frac{r-2}{r})$	...	$(\frac{r-i+1}{r}, \frac{r-i}{r})$	$(\frac{r-i}{r}, \frac{r-i-1}{r})$	...	$(\frac{r-i}{r}, \frac{1}{0})$
	Δ_1	Δ_2	...	Δ_i	Δ_{i+1}	...	Δ_r

步骤7 o_i

3.4.2 自然灾害等级评价实例分析及结论

2010 2002-2009

3-2 3-3

f

g *f* *g* 8 *f*

2002 2003 2004 2005 2006 2007 2008 2009*g*

表 3-2 全国历年自然灾害情况(2002年-2009年)

2002	40246	2759	509740	5	362	13100
2003	15489	1333	504325	21	7465	466040
2004	13555	1407	408828	11	696	94959
2005	17751	1223	357678	13	882	262811
2006	102804	1227	431590	10	7229	79962
2007	25364	1123	247528	3	422	201922
2008	26580	1598	326936	17	446293	85949594
2009	10580	845	190109	8	407	273782

表 3-3 续表-全国历年自然灾害情况(2002年-2009年)

2002	126	124	65.9	7527	98	3610
2003	172	128	80.5	10463	142	37000
2004	155	140	54.2	13466	252	20213
2005	176	371	332.4	11542	152	15029
2006	180	492	218.5	8170	102	5375
2007	163	161	88.4	3	94	12416
2008	128	152	206.1	14144	174	12594
2009	132	95	100.2	8859	110	14511

步骤1 3-2 3-3 3-6 $\bar{\ } = 0.6$

3-4,

表 3-4 自然灾害评价矢量

2002	0.648	1.000	1.000	0.36	0.541	0.541	0.169	0.481	0.436	0.287	0.447	0.354
2003	0.509	0.567	0.984	1.00	0.549	0.544	0.876	0.487	0.467	0.603	0.605	1.000
2004	0.499	0.590	0.706	0.60	0.542	0.542	0.615	0.504	0.411	0.927	1.000	0.675
2005	0.522	0.534	0.558	0.68	0.542	0.543	0.938	0.829	1.000	0.719	0.641	0.575
2006	1.000	0.535	0.772	0.56	0.541	0.542	1.000	1.000	0.759	0.356	0.461	0.575
2007	0.565	0.504	0.238	0.28	0.541	0.542	0.738	0.534	0.483	0.474	0.433	0.524
2008	0.572	0.648	0.468	0.84	1.000	1.000	0.200	0.521	0.732	1.000	0.720	0.528
2009	0.482	0.419	0.071	0.48	0.541	0.543	0.261	0.441	0.508	0.431	0.490	0.565

步骤2

3-1 3-2 $w_p=0.5991$ $w_e=0.3252$ $w_t=0.0757$

$$W_2=(0.3252,0.0757,\ 0.5991)$$

3-1

3-2 $w_{zh}=0.4000$ $w_z=0.3414$ $w_h=0.2000$ $w_s=0.0586$

$$W_1=(0.3414,\ 0.4000,\ 0.2000,\ 0.0586)$$

步骤3

$$T_2=\begin{pmatrix} 0.648 & 0.360 & 0.169 & 0.287 \\ 1.000 & 0.541 & 0.481 & 0.447 \\ 1.000 & 0.541 & 0.436 & 0.354 \end{pmatrix};$$

步骤4 2003

$$V_{2003}=W_2T_2=(0.3252;0.0757;0.5991)\begin{pmatrix} 0.648 & 0.360 & 0.169 & 0.287 \\ 1.000 & 0.541 & 0.481 & 0.447 \\ 1.000 & 0.541 & 0.436 & 0.354 \end{pmatrix}$$

$$=(0.9734;0.5281;0.4310;0.3799).$$

步骤5 2004-2009

T

$$T=\begin{pmatrix} 0.9734 & 0.8129 & 0.6532 & 0.5477 & 0.7129 & 0.3493 & 0.5349 & 0.2157 \\ 0.5281 & 0.5803 & 0.5466 & 0.5532 & 0.5434 & 0.5226 & 0.9879 & 0.5380 \\ 0.4310 & 0.5048 & 0.4572 & 0.9400 & 0.8557 & 0.5195 & 0.6237 & 0.4681 \\ 0.3799 & 0.8417 & 0.8001 & 0.6078 & 0.5219 & 0.4912 & 0.6265 & 0.5309 \end{pmatrix};$$

$$O = W_1 T = (0.3414, 0.4000, 0.2000, 0.0586)\begin{pmatrix} 0.9734 & 0.8129 & 0.6532 & 0.5477 & 0.7129 & 0.3493 & 0.5349 & 0.2157 \\ 0.5281 & 0.5803 & 0.5466 & 0.5532 & 0.5434 & 0.5226 & 0.9879 & 0.5380 \\ 0.4310 & 0.5048 & 0.4572 & 0.9400 & 0.8557 & 0.5195 & 0.6237 & 0.4681 \\ 0.3799 & 0.8417 & 0.8001 & 0.6078 & 0.5219 & 0.4912 & 0.6265 & 0.5309 \end{pmatrix}$$
$$= (0.6520, 0.6600, 0.5799, 0.6319, 0.6625, 0.4610, 0.7392, 0.4236).$$

步骤6 ¢ = f

g 3-5

表 3-5 2002-2009自然灾害等级评价结果

	1	2	3	4	5	6
	(1, 0.83)	(0.83, 0.67)	(0.67, 0.5)	(0.5, 0.33)	(0.33, 0.17)	(0.17,0)

步骤7 3-5 O 08 02 03 04 05 06 07 09

3.5 本章小结

3.1

2002-2009

08 02 03 04 05 06 07 09

4 基于属性二元关系偏好的多属性大群体决策方法

De Morgan

[73]

0-1 [74] [75]

[76]

[77]

4.1 决策问题研究背景

[78]

[79]

[65]

MADM

MADM [80]-[82]

MADM [83]-[85] MADM [86]

[87]

ELECTRE Sugen

0-1

4.2 基于属性二元关系的偏好矢量间相似度

定义4.1 $X=(x_1,x_2,\ldots,x_n)$ $Y=(y_1,y_2,\ldots,y_n)$ R n , $S(X,Y)$ $S(X,Y)$ X Y

$$(P1)\ 0 \le S(X,Y) \le 1; \tag{4-1}$$

$$(P2)\ X=Y,\ x_i=y_i,\ i=1,2,\ldots,n,\ S(X,Y)=1; \tag{4-2}$$

$$(P3)\ S(X,Y)=S(Y,X). \tag{4-3}$$

Q X $Q(x_1,x_2)$, $Q(x_1,x_2)$ $X\times X \to [0,1]$ $Q(x_1,x_2)$ x_1 x_2 $1\le i,j\le n$ $T(Q)=(t_{ij})_{n\times n}$ X Q $t_{ij}=1$ $(x_i,x_j)\in Q$ $t_{ij}=0$ $T(Q)$ 0-1 $X=(0.2,0.4,0.8)$ Q x_i x_j $(i=1,2,3;j=1,2,3)$ 1 X Q :

$$T(Q)=\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}.$$

定义 4.2 ^) Ðï!ò _ V€ÐÔð

$T(Q_1)$ Q - 1 0TD[005 12.005Tf 12.005 0TD[(:)]TJET(9(4.2)

$$\|T\|_2 = (\rho(T'T))^{\frac{1}{2}}; \quad \|T_i\|_\infty = \max_{1\le j_1\le n} \sum_{j=1} |t_{ij}|$$

$\rho(T'T)$ $T'T$ $T'T$

定理 4.1. **对于定义***4.3***中两个矢量**X_i **和**X_j **基于二元关系**Q **的相似度，满足定义***4.1***中的性质，是两个矢量之间的相似度。**

证明. (P1) $S_{ij}(X_i;X_j) \ge 0$ $S_{ij}(X_i;X_j) \le 1$

$$\|T\|_2 \le \sqrt{n}\,\|T\|_\infty; \; \|T_1+T_2\|_2 \le \|T_1\|_2 + \|T_2\|_2; \tag{4-6}$$

4-6 4-5

$$\begin{aligned} S_{ij}(X_i;X_j) &= \frac{1}{\sqrt{n}} \times \frac{\|T_i+T_j\|_2}{\|T_i\|_\infty + \|T_j\|_\infty} \le \frac{1}{\sqrt{n}} \times \frac{\|T_i\|_2 + \|T_j\|_2}{\|T_i\|_\infty + \|T_j\|_\infty} \\ &\le \frac{\sqrt{n}}{\sqrt{n}} \times \frac{\|T_i\|_\infty + \|T_j\|_\infty}{\|T_i\|_\infty + \|T_j\|_\infty} \le 1. \end{aligned}$$

(P2) 0-1 $\|T\|_2 = \sqrt{n}\,\|T\|_\infty$

$$S_{ij}(X_i;X_i) = \frac{1}{\sqrt{n}} \frac{\|T_i+T_i\|_2}{\|T_i\|_\infty + \|T_i\|_\infty} = \frac{1}{\sqrt{n}} \frac{2\sqrt{n}\|T_i\|_\infty}{2\|T_i\|_\infty} = 1.$$

(P3)

$$S_{ij}(X_i;X_j) = \frac{1}{\sqrt{n}} \frac{\|T_i+T_j\|_2}{\|T_i\|_\infty + \|T_j\|_\infty} = S_{ji}(X_j;X_i)$$

□

4.3 基于属性二元关系的大群体聚类算法

p $A = \{A_1;A_2;...;A_p\}$ n $C = \{C_1;C_2;...;C_n\}$ m - p n i l v_j^i $v_j^i \ge 0$ $i = 1;2;...;m; j = 1;2;...;n$ $V^i = (v_1^i;v_2^i;...v_n^i)$ - i

定义 4.4 . i V^i Q n $V^i = (v_1^i;v_2^i;...v_n^i)$ $0 \le j_1;j_2 \le 1$ $T^i(Q) = (t^i_{j_1 j_2})_{n\times n}$ i Q

$t^i_{j_1 j_2} = 1$ $(v^i_{j_1}; v^i_{j_2}) \ 2 \ Q$ $v^i_{j_1}; v^i_{j_2}$
Q $t^i_{j_1 j_2} = 0$ $T^i(Q)$ 0-1
4.3

T^i Q 4-5

棰 量 E (

4.3.1 属性关系矩阵 $T = fT^ig$ 形成

V^i $T = fT^ig$

步骤1 - a
1 » m V^i

步骤2 Q

步骤3 i=1

步骤4 a $V^i = (v^i_1; v^i_2; :::v^i_n)$ $V^i \ 2$ a
T

步骤5 a V^i $(v^i_{j_1}; v^i_{j_2}) \ 2 \ Q$
Q $t^i_{j_1 j_2} = 1$ $t^i_{j_1 j_2} = 0$ $(j_1; j_2$=1,2,...,n)
$T^i(Q) = (t^i_{j_1 j_2})_{n \times n}$ $T^i(Q)$ T 成金炎 VŸ

步骤6 a a V^i $(i = i + 1)$
$V^i \ 2$ a 5 a 7

步骤7

C^k

$$H^k = \frac{h^k}{|h^k|}, \tag{4-8}$$

$|h^k|$ C^k -

$$U = \frac{u}{|u|}, \tag{4-9}$$

1.2 4.2.1

20 20

$T = \{T^1; T^2; ...; T^{20}\}$ 1 2 4

$$T^1(Q_1) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}; T^2(Q_1) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix};$$

$$T^4(Q_1) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}:$$

γ=0.75 20 $T = \{T^1$, T^2, ..., $T^{20}\}$ 4.3.2 [88] K=4 4-2

表 4-2 基于二元关系Q_1，γ=0.75时的聚类算法执行结果

C^1	13	$V^1; V^4; V^5; V^6; V^7; V^8; V^9; V^{11}; V^{16}; V^{17}; V^{18}; V^{19}; V^{20}$
C^2	4	$V^2; V^{10}; V^{12}; V^{15}$
C^3	2	$V^3; V^{13}$
C^4	1	V^{14}

4-5 1 2 $r_{12}(V^1; V^2) = 0 < 0.75$ 2 1 4 $r_{14}(V^1; V^4) = 1 > 0.75$ 1 4 C^1

γ =0.9 K=7 4-3

表 4-3 基于二元关系Q_1，γ=0.9时的聚类算法执行结果

C^1	10	$V^1; V^4; V^5; V^6; V^8; V^9; V^{16}; V^{17}; V^{19}; V^{20}$
C^2	2	$V^2; V^7$
C^3	3	$V^3; V^{10}; V^{12}$
C^4	2	$V^{11}; V^{18}$
C^5	1	V^{13}
C^6	1	V^{14}
C^7	1	V^{15}

2 Q_2 $\log(x_i + x_j)$ ¸

$0{:}1$ $(x_i; x_j)\ 2\ Q_2$

0.1 ° =0.75 K=7 4-4

表 4-4 基于二元关系Q_2，γ=0.75时的聚类算法执行结果

C^1	10	$V^1; V^2; V^3; V^6; V^9; V^{17}; V^{18}; V^{19}; V^{20}$
C^2	2	$V^4; V^{15}$
C^3	4	$V^5; V^8; V^{10}; V^{14}$
C^4	1	V^7
C^5	1	V^{11}
C^6	1	V^{12}
C^7	1	V^{13}

° =0.9 K=14 4-5

表 4-5 基于二元关系Q_2，γ=0.9时的聚类算法执行结果

C^1	5	$V^1; V^2; V^3; V^6; V^7$
C^2	1	V^4
C^3	3	$V^5; V^{14}; V^{16}$
C^4	1	V^7
C^5	1	V^8
C^6	1	V^9
C^7	1	V^{10}
C^1	1	V^{11}
C^2	1	V^{12}
C^3	1	V^{13}
C^4	1	V^{15}
C^5	1	V^{18}
C^6	1	V^{19}
C^7	1	V^{20}

4-2 - 4-5 Q

Q

Q

4.4.2 不同的聚类方法的算例结果比较

[88]

定义 4.5: [88]. -

i C^i(

ρ^i

$$\rho^i = \frac{1}{C_{n_i}^2}\sum_{i_1,i_2=1}^{n_i} r_{i_1 i_2}(V^{i_1},V^{i_2}), \quad (4\text{-}10)$$

$V^{i_1},V^{i_2} \in C^i$ $(i_1,i_2 = 1,2,\ldots,n_i)$ $C_{n_i}^2 = \frac{n_i(n_i-1)}{2}$

定义 4.6 [88].

$$\rho = \sum_{i=1}^{k}\frac{n_i}{m}\cdot\rho^i, \quad (4\text{-}11)$$

$\sigma = 1 - \rho$ ρ

[88] γ=0.75 γ=0.9

4-1 [88]

4-10 4-11

4-6

表 4-6 不同聚类方法的聚类结果比较

[88]	γ=0.75	ρ=0.690	σ=0.310
[88]	γ=0.90	$\rho = 0.687$	σ=0.313
	γ=0.75	ρ=0.840	σ=0.160
	γ=0.90	ρ=0.750	σ=0.250

4-6 [88]

4.5 基于属性二元关系偏好的决策方案排序

4.5.1 基于属性二元关系的决策属性权重

[89] [90]

[91] Q

定义4.7 . i V^i n
$V^i = (v_1^i, v_2^i, \ldots, v_n^i)$ Q $T^i(Q)$ i
Q Q

$$T(Q) = T^1(Q) \oplus T^2(Q) \oplus \ldots \oplus T^m(Q). \tag{4-12}$$

$T(Q)$ 0-1
[2] W Q

$$T(Q) \cdot W = \lambda_{Max} \cdot W. \tag{4-13}$$

$$T(Q) = \begin{pmatrix} t_{11} & t_{12} & \ldots & t_{1n} \\ t_{21} & t_{22} & \ldots & t_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ t_{n1} & t_{n2} & \ldots & t_{nn} \end{pmatrix},$$

$W = (w_1, w_2, \ldots, w_n)^T$ λ_{Max} $T(Q)$ W λ_{Max}

4.5.2 基于属性二元关系的决策方案排序原则

m Q $ª =$

$\{V^i\}$ K 4-9

$U = (u_1, u_2, \cdots, u_n)$ P $A =$

$\{A_1, A_2, \cdots, A_p\}$ $A_l($ $l = 1, 2, \cdots, p)$

$U^l = (u_1^l, u_2^l, \cdots, u_n^l)^T$,

$$T^U = (U^1, U^2, \cdots, U^p) = \begin{pmatrix} u_1^1 & u_1^2 & \cdots & u_1^p \\ u_2^1 & u_2^2 & \cdots & u_2^p \\ \vdots & \vdots & \vdots & \vdots \\ u_n^1 & u_1^n & \cdots & u_n^p \end{pmatrix}, \tag{4-14}$$

$$O = W^T \cdot T^U = (w_1, w_2, \cdots, w_n) \cdot \begin{pmatrix} u_1^1 & u_1^2 & \cdots & u_1^p \\ u_2^1 & u_2^2 & \cdots & u_2^p \\ \vdots & \vdots & \vdots & \vdots \\ u_n^1 & u_1^n & \cdots & u_n^p \end{pmatrix} = (o_1, o_2, \cdots, o_p), \tag{4-15}$$

O

4.6 基于属性二元关系的偏好相似度在应急决策中的应用

[63]

5

30

- [63] 61 20

6 61

6

Q

$log(x_i + x_j) \geq 0.5$ $(x_i, x_j) \in Q$

0.5

6

1 2 3 4 5 6

5 $\{A_1, A_2, A_3, A_4, A_5\}$

30

5 30 $\{V^i, i = 1, 2, \cdots, 30\}$,

$\{V^{1} \sim V^{6}\}$ $\{V^{7} \sim V^{13}\}$ $\{V^{14} \sim V^{18}\}$ $\{V^{19} \sim V^{24}\}$ $\{V^{25} \sim V^{30}\}$

0 1

[2]

$$Z = \frac{y_j^{max} - y_{ij}}{y_j^{max} - y_j min};$$

$\Omega = \{V^{i} | i = 1, 2, \ldots, 30\}$ 4-7

步骤1 γ =0.9 4-5 Q

$r_{ij}(X_i, X_j) = \frac{1}{\sqrt{n}} \frac{\|T_i + T_j\|_2}{\|T_i\|_1 + \|T_j\|_1}$

4-8

步骤2 4-8

$$T^{U} = (U^{1}, U^{2}, \ldots, U^{6}) = \begin{pmatrix} 0.44 & 0.39 & 0.39 & 0.25 & 0.40 \\ 0.47 & 0.51 & 0.44 & 0.44 & 0.34 \\ 0.33 & 0.39 & 0.34 & 0.45 & 0.51 \\ 0.33 & 0.44 & 0.31 & 0.49 & 0.40 \\ 0.41 & 0.28 & 0.36 & 0.35 & 0.36 \\ 0.45 & 0.38 & 0.53 & 0.39 & 0.38 \end{pmatrix}.$$

步骤3 4-7 4.4 Q

$T^{i}(Q)$ $\{T^{i}(Q) | i = 1, 2, \ldots, 30\}$

4-10 :

$$T(Q) = T^{1}(Q) \oplus T^{2}(Q) \oplus \cdots \oplus T^{30}(Q) = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \end{pmatrix};$$

4-11 W^{T}=(0.2467, 0.1680, 0.1374, 0.0031, 0.2761, 0.1732)

步骤4 4-13 5

表 4-7 群体成员偏好矢量

	1	2	3	4	5	6
V^1	0.8452	0.9037	1.0000	1.0000	0.8003	0.9316
V^2	0.4410	0.6640	0.4722	0.6227	0.5933	0.6010
V^3	0.8613	0.7228	0.7569	0.3666	0.7118	0.9731
V^4	1.0000	0.0000	0.3151	0.1609	0.5933	0.0432
V^5	0.6810	0.9374	0.6152	0.8043	0.7031	1.0000
V^6	0.1113	1.0000	0.0000	0.1963	0.3100	0.5725
V^7	0.6206	0.7769	0.4711	0.5180	0.3172	0.4194
V^8	0.9055	0.9256	0.6170	0.8522	1.0000	0.7818
V^9	1.0000	0.8582	0.7363	0.8865	0.5421	0.8721
V^{10}	0.9786	0.7769	0.8008	0.7156	0.5039	0.4552
V^{11}	0.3351	0.2397	0.2104	0.2844	0.2282	0.2720
V^{12}	0.1273	0.6164	0.4122	0.3690	0.1098	0.4389
V^{13}	0.0000	0.2996	0.3252	0.4216	0.1683	0.2370
V^{14}	0.4397	0.4730	0.3241	0.3034	0.4352	0.6344
V^{15}	0.5536	0.6292	0.6152	0.6836	0.7534	0.8070
V^{16}	0.3338	0.0813	0.4122	0.1254	0.1098	0.6344
V^{17}	0.8311	0.8716	0.4711	0.6387	0.5933	0.6360
V^{18}	0.3847	0.8716	0.3316	0.3034	0.4352	0.6344
V^{19}	0.3143	0.5832	0.2989	0.5299	0.5164	0.5081
V^{20}	0.4028	0.4724	0.5855	0.6682	0.2282	0.2655
V^{21}	0.1709	0.5211	0.5311	0.4885	0.6750	0.4585
V^{22}	0.3787	0.3590	0.5821	0.6943	0.2282	0.4503
V^{23}	0.0657	0.0958	0.4122	0.0875	0.0000	0.1319
V^{24}	0.2145	0.9609	0.4122	0.7516	0.7273	0.8241
V^{25}	0.3532	0.2397	0.5049	0.0000	0.2969	0.0000
V^{26}	0.4584	0.4318	0.6459	0.7516	0.8090	0.9560
V^{27}	0.2862	0.2397	0.5345	0.5186	0.2282	0.1792
V^{28}	0.3311	0.4318	0.4722	0.6138	0.5933	0.7085
V^{29}	0.5938	0.6956	0.6197	0.0124	0.4836	0.4568
V^{30}	0.5724	0.3371	0.4122	0.7167	0.2282	0.6344

表 4-8 基于关系Q，γ =0.9时的群体成员聚类结果

n_k		H^k	E
2	$V^1; V^2$	(0.35,0.43,0.40,0.45,0.38,0.42)	(0.44, 0.47, 0.33,
1	V^6	(0.09,0.82,0.00,0.16,0.26,0.47)	0.33,0.41,0.45)
2	$V^3; V^4$	(0.65,0.25,0.37,0.18,0.45,0.35)	
1	V^5	(0.37,0.48,0.31,0.41,0.36,0.51)	
4	$V^7; V^{11}$	(0.31,0.56,0.41,0.45,0.23,0.39)	(0.39,0.51,0.39,
	$V^{12}; V^{13}$		
1	V^8	(0.43,0.44,0.29,0.40,0.47,0.37)	0.44,0.28,0.38)
1	V^9	(0.49,0.4,0.36,0.43,0.26,0.42)	
1	V^{10}	(0.54,0.43,0.44,0.40,0.28,0.25)	
3	$V^{14}; V^{16}; V^{15}$	(0.37,0.33,0.38,0.31,0.37,0.59)	(0.39,0.44,0.34,
1	V^{17}	(0.49,0.51,0.27,0.37,0.35,0.37)	0.31,0.36,0.53)
1	V^{18}	(0.29,0.66,0.25,0.23,0.33,0.48)	
5	$V^{19}; V^{20}; V^{21}$	(0.27,0.41,0.49,0.50,0.33,0.37)	
	$V^{22}; V^{23}$		(0.25,0.44,0.45,
1	V^{24}	(0.12,0.56,0.24,0.44,0.42,0.48)	0.49,0.35,0.39)
6	$V^{25}; V^{26}; V^{27}$	(0.40,0.34,0.51,0.40,0.36,0.38)	(0.40,0.34,0.51,
	$V^{28}; V^{29}; V^{30}$		0.40,0.36,0.38)

$$O = W^T \cdot T^U = (0.2467, 0.1680, 0.1374, 0.0031, 0.2761, 0.1732)$$

$$\begin{pmatrix} 0.44 & 0.39 & 0.39 & 0.25 & 0.40 \\ 0.47 & 0.51 & 0.44 & 0.44 & 0.34 \\ 0.33 & 0.39 & 0.34 & 0.45 & 0.51 \\ 0.33 & 0.44 & 0.31 & 0.49 & 0.40 \\ 0.41 & 0.28 & 0.36 & 0.35 & 0.36 \\ 0.45 & 0.38 & 0.53 & 0.39 & 0.38 \end{pmatrix} = (0.4243, 0.3840, 0.4150, 0.367, 0.3988).$$

0.4243

½[88] ½=0.6546

[63] ½=0.5964

[63]

4.7 本章小结

n ! 1

$r(x;y)$! 1 n

5 基于模糊集偏好的多属性大群体决策方法

[92]-[93]

[94] Zadeh[20] 1965

Atanassov[25]

J- E- C-

J- E-

C- E-

5.3 [31] 13

5.4 E-

5.1 模糊集的概念及两模糊集间的相似度

(Fuzzy Set, FS)

$X = \{x_1, x_2, \ldots, x_n\}$, X

$\Omega(X)$ $W = (w_1, w_2, \ldots, w_n)$ $x_j (j = 1, 2, \ldots, n)$,

$w_j \geq 0$ $\sum_{j=1} w_j = 1$

Zadeh [20]

定义5.1 $X = \{x_1, x_2, \ldots, x_n\}$ X

$$A = \{< x_j, \mu_A(x_j) > | x_j \in X\},$$

$\mu_A(x) : x \to [0, 1]$ $\mu_A(x_j)$ X x_j A

A B X $A = \{< x_j, \mu_A(x_j) > | x_j \in X\}$ $B = \{< x_j, \mu_B(x_j) > | x_j \in X\}$ A B

定义5.2 A B J- E- C-

$$S_J^{FS}(A,B) = \frac{1}{n} \frac{\sum_{i=1} \mu_A(x_i)\mu_B(x_i)}{\sum_{i=1} \mu_A^2(x_i) + \sum_{i=1} \mu_B^2(x_i) - \sum_{i=1} \mu_A(x_i)\mu_B(x_i)}, \tag{5-1}$$

$$S_E^{FS}(A,B) = \frac{1}{n} \frac{2\sum_{i=1} \mu_A(x_i)\mu_B(x_i)}{\sum_{i=1} \mu_A^2(x_i) + \sum_{i=1} \mu_B^2(x_i)}, \tag{5-2}$$

$$S_C^{FS}(A,B) = \frac{1}{n} \frac{\sum_{i=1} \mu_A(x_i)\mu_B(x_i)}{\sqrt{\sum_{i=1} \mu_A^2(x_i)}\sqrt{\sum_{i=1} \mu_B^2(x_i)}}. \tag{5-3}$$

定理 5.1. **定义*5.2*中的两个模糊集之间的三种相似度满足下列性质：**

$(P1)$ $0 \le S^{FS}(A,B) \le 1$;

$(P2)$ $S^{FS}(A,B) = S_V^{FS}(B,A)$;

$(P3)$ $S^{FS}(A,B) = 1, A = B, \mu_A(x_i) = \mu_B(x_i),\ i = 1,2,\ldots,n.$

证明. $S_J^{FS}(A,B)$ 5.1

(P1) $S_J^{FS}(A,B) \ge 0$ $S_J^{FS}(A,B) \le 1$.

$$\sum_{i=1} \mu_A^2(x_i) + \sum_{i=1} \mu_B^2(x_i) \ge 2\sum_{i=1} \mu_A(x_i)\mu_B(x_i).$$

$$S_J^{FS}(A,B) = \frac{1}{n} \frac{\sum_{i=1} \mu_A(x_i)\mu_B(x_i)}{\sum_{i=1} \mu_A^2(x_i) + \sum_{i=1} \mu_B^2(x_i) - \sum_{i=1} \mu_A(x_i)\mu_B(x_i)} \le \frac{1}{n}\sum_{i=1} 1 \le 1.$$

(P2) $S_J^{FS}(A,B) = S_J^{FS}(B,A)$

(P3) $A = B$, $\mu_A(x_i) = \mu_B(x_i)$, $i = 1,2,\ldots,n$

$$S_J^{FS}(A,B) = \frac{1}{n} \frac{\sum_{i=1} \mu_A(x_i)\mu_A(x_i)}{\sum_{i=1} \mu_A^2(x_i) + \sum_{i=1} \mu_A^2(x_i) - \sum_{i=1} \mu_A(x_i)\mu_A(x_i)} = 1.$$

$S_E^{FS}(A;B)$ $S_C^{FS}(A;B)$

□

定义5.3 $A=\{\mu_A(x_1);\mu_A(x_2),\ldots;\mu_A(x_n)\}$ $B=\{\mu_B(x_1),\mu_B(x_2),\ldots,\mu_B(x_n)\}$ $X=\{x_1;x_2;\ldots;x_n\}$, $x_i\in X$

$$S_V^{FS}(A;B)=\alpha\frac{2AB}{\|A\|_2^2+\|B\|_2^2}+(1-\alpha)\frac{AB}{\|A\|_2\|B\|_2}; \tag{5-4}$$

$AB=\sum_{i=1}\mu_A(x_i)\mu_B(x_i)$ $\|A\|_2=\sqrt{\sum_{i=1}\mu_A^2(x_i)}$ $\|B\|_2=\sqrt{\sum_{i=1}\mu_B^2(x_i)}$ $0\le\alpha\le1$

[24] 5.2

定理 5.2. 两个模糊集之间的变系数相似度满足下列性质：

$$(P1)\ 0\le S_V^{FS}(A;B)\le1; \tag{5-5}$$

$$(P2)\ S_V^{FS}(A;B)=S_V^{FS}(B;A); \tag{5-6}$$

$$(P3)\ S_V^{FS}(A;B)=1; A=B; \mu_A(x_i)=\mu_B(x_i);\quad i=1;2;\ldots;n. \tag{5-7}$$

证明. (P1) $S_V^{FS}(A;B)\ge0$ $S_V^{FS}(A;B)\le1$

$$0\le\frac{2AB}{\|A\|_2^2+\|B\|_2^2}\le1; 0\le\frac{AB}{\|A\|_2\|B\|_2}\le1. \tag{5-8}$$

(5-8) (5-4)

$$S_V^{FS}(A;B)\le\alpha+(1-\alpha)=1. \tag{5-9}$$

(P2) $S_V^{FS}(A;B)=S_V^{FS}(B;A)$

(P3) $A=B$, $\mu_A(x_i)=\mu_B(x_i)$, $i=1;2;\ldots;n$

$$\begin{aligned}S_V^{FS}(A;B)&=\alpha\frac{2AA}{\|A\|_2^2+\|A\|_2^2}+(1-\alpha)\frac{AA}{\|A\|_2\|A\|_2}\\&=\alpha\frac{2\sum_{i=1}\mu_A(x_i)\mu_A(x_i)}{\sum_{i=1}\mu_A^2(x_i)+\sum_{i=1}\mu_A^2(x_i)}+(1-\alpha)\frac{\sum_{i=1}\mu_A(x_i)\mu_A(x_i)}{\sqrt{\sum_{i=1}\mu_A^2(x_i)}\sqrt{\sum_{i=1}\mu_A^2(x_i)}}\\&=\alpha+1-\alpha=1.\end{aligned}$$

□

5.2 直觉模糊集的概念及两直觉模糊集间的相似度

[24],[95] Li Cheng[26]

Liang Shi[27] 2003 [24]

Szmidt Kacprzyk[96]-[97]

Huang Yang[

A ¼$_A(x_j)$ = 1 ¡ ¹$_A(x_j)$ ¡ º$_A(x_j)$, x_j 2 X, ¼$_A(x_j)$
x_j X Szmiet Kacprzyk ¼$_A(x_j)$ x_j X
, ¼$_A(x_j)$ = 0, x_j 2 X A

5.2.2 基于直觉模糊集的变系数相似度模型

定义5.5 s S : -(X)^2 ! [0; 1] :X

, $0 \le S_E(A,B) \le 1$

(P2) $A = B$, $\mu_A(x_i) = \mu_B(x_i)$ $\nu_A(x_i) = \nu_B(x_i)$ $i = 1,2,\ldots,n$.

$$S_E(A,B) = \frac{1}{n}\sum_{i=1} \frac{2(\mu_A(x_i)\mu_A(x_i) + \nu_A(x_i)\nu_A(x_i))}{(\mu_A^2(x_i) + \nu_A^2(x_i)) + (\mu_A^2(x_i) + \nu_A^2(x_i))} = 1.$$

□

J-

定义5.7 $A = \{<x_j, \mu_A(x_j), \nu_A(x_j)> | x_j \in X\}$ $B = \{<x_j, \mu_B(x_j), \nu_B(x_j)> | x_j \in X\}$, X , $A, B \in \Phi(X)$ A B

$$S_V(A,B) = \alpha\frac{1}{n}\sum_{i=1} \frac{2(\mu_A(x_i)\mu_B(x_i) + \nu_A(x_i)\nu_B(x_i))}{(\mu_A^2(x_i) + \nu_A^2(x_i)) + (\mu_B^2(x_i) + \nu_B^2(x_i))}$$
$$+(1-\alpha)\frac{1}{n}\sum_{i=1} \frac{\mu_A(x_i)\mu_B(x_i) + \nu_A(x_i)\nu_B(x_i)}{\sqrt{\mu_A^2(x_i) + \nu_A^2(x_i)}\sqrt{\mu_B^2(x_i) + \nu_B^2(x_i)}}. \quad (5\text{-}15)$$

$0 \le \alpha \le 1$ $\alpha = 0$ [31]

$\alpha = 1$ 5.6 $E-$

5.7

定理 5.4. $S_V(A,B)$ **是两个直觉模糊集之间的相似度，且**A **和**$B \in \Phi(X)$.

证明. $S_V(A,B)$ $S_V(A,B) = S_V(B,A)$ $S_V(A,B)$ (P1-P2)

(P1) $S_V(A,B) \ge 0$ $S_V(A,B) \le 1$

$$\mu_A^2(x_i) + \mu_B^2(x_i) \ge 2\mu_A(x_i)\mu_B(x_i), \quad \nu_A^2(x_i) + \nu_B^2(x_i) \ge 2\nu_A(x_i)\nu_B(x_i),$$
$$\mu_A(x_i)\mu_B(x_i) + \nu_A(x_i)\nu_B(x_i) \le \sqrt{\mu_A^2(x_i) + \nu_A^2(x_i)}\sqrt{\mu_B^2(x_i) + \nu_B^2(x_i)},$$

5-15

$$S_V(A,B) = \alpha\frac{1}{n}\sum_{i=1} \frac{2(\mu_A(x_i)\mu_B(x_i) + \nu_A(x_i)\nu_B(x_i))}{(\mu_A^2(x_i) + \nu_A^2(x_i)) + (\mu_B^2(x_i) + \nu_B^2(x_i))}$$
$$+(1-\alpha)\frac{1}{n}\sum_{i=1} \frac{\mu_A(x_i)\mu_B(x_i) + \nu_A(x_i)\nu_B(x_i)}{\sqrt{\mu_A^2(x_i) + \nu_A^2(x_i)}\sqrt{\mu_B^2(x_i) + \nu_B^2(x_i)}}$$
$$\le \alpha + 1 - \alpha \le 1.$$

$0 \cdot S_V(A;B) \cdot 1$

(P2) $A = B$, $¹_A(x_i) = ¹_B(x_i)$ $º_A(x_i) = º_B(x_i)$ $i = 1;2;:::;n$.

$$S_V(A;B) = ®\frac{1}{n}\sum_{i=1}^{n}\frac{2(¹_A(x_i)¹_A(x_i) + º_A(x_i)º_A(x_i))}{(¹_A^2(x_i) + º_A^2(x_i)) + (¹_A^2(x_i) + º_A^2(x_i))}$$
$$+(1 ¡ ®)\frac{1}{n}\sum_{i=1}^{n}\frac{¹_A(x_i)¹_A(x_i) + º_A(x_i)º_A(x_i)}{\sqrt{¹_A^2(x_i) + º_A^2(x_i)}\sqrt{¹_A^2(x_i) + º_A^2(x_i)}}$$
$$= 1:$$

□

$x_j (j = 1;2;:::;n; x_j \ 2 \ X)$ 5-15

$x_j \ (j = 1;2;:::;n)$

$$WS_V(A;B) = \sum_{i=1}^{n} w_i[®\frac{2(¹_A(x_i)¹_B(x_i) + º_A(x_i)º_B(x_i))}{¹_A^2(x_i) + º_A^2(x_i) + ¹_B^2(x_i) + º_B^2(x_i)}$$

$+(1 ¡ ®)$ 7-1.801TD[(A)]TJ/F312./F2J/F71.97Tf7.0724TD[( )

5.3 不同的两直觉模糊集之间的相似度的比较

A B $X = \{x_1, x_2, \ldots, x_n\}$ [31]

13

$T_A(x_i) = \mu_A(x_i) - \nu_A(x_i)$ $T_B(x_i) = \mu_B(x_i) - \nu_B(x_i)$ $T_\mu(x_i) = \mu_A(x_i) - \mu_B(x_i)$ $T_\nu(x_i) = \nu_A(x_i) - \nu_B(x_i)$

$$S_C(A,B) = 1 - \frac{\sum_{i=1}^n |T_A(x_i) - T_B(x_i)|}{2n}, \quad (5\text{-}19)$$

$$S_H(A,B) = 1 - \frac{\sum_{i=1}^n |T_\mu(x_i)| + |T_\nu(x_i)|}{2n}, \quad (5\text{-}20)$$

$$S_L(A,B) = 1 - \frac{\sum_{i=1}^n |T_A(x_i) - T_B(x_i)|}{4n} - \frac{\sum_{i=1}^n |T_\mu(x_i)| + |T_\nu(x_i)|}{4n}, \quad (5\text{-}21)$$

$$S_O(A,B) = 1 - \sqrt{\frac{\sum_{i=1}^n (T_\mu^2(x_i) + T_\nu^2(x_i))}{2n}}, \quad (5\text{-}22)$$

$$S_{DC}(A,B) = 1 - \sqrt[p]{\frac{\sum_{i=1}^n |\psi_A(x_i) - \psi_B(x_i)|^p}{n}}, \quad (5\text{-}23)$$

$\psi_A(x_i) = (\mu_A(x_i) + 1 - \nu_A(x_i))/2$ $\psi_B(x_i) = (\mu_B(x_i) + 1 - \nu_B(x_i))/2;$

$$S_{HB}(A,B) = \frac{\rho_\mu(A,B) + \rho_\nu(A,B)}{2}, \quad (5\text{-}24)$$

$\rho_\mu(A,B) = 1 - \sqrt[p]{\frac{\sum_{i=1}^n |\mu_A(x_i) - \mu_B(x_i)|^p}{n}}$ $\rho_\nu(A,B) = 1 - \sqrt[p]{\frac{\sum_{i=1}^n |\nu_A(x_i) - \nu_B(x_i)|^p}{n}}$

$S_{HB}(A,B) = S_H(A,B)$ p=1;

$$S_e^p(A,B) = 1 - \sqrt[p]{\frac{\sum_{i=1}^n (\phi_\mu(x_i) + \phi_\nu(x_i))^p}{n}}, \quad (5\text{-}25)$$

$\phi_\mu(x_i) = |\mu_A(x_i) - \mu_B(x_i)|/2$ $\phi_\nu(x_i) = |(1 - \nu_A(x_i))/2 - (1 - \nu_B(x_i))/2|;$

$$S_s^p(A,B) = 1 - \sqrt[p]{\frac{\sum_{i=1}^n (\varphi_{s1}(x_i) + \varphi_{s2}(x_i))^p}{n}}, \quad (5\text{-}26)$$

$\varphi_{s1}(x_i) = |m_{A1}(x_i) - m_{B1}(x_i)|/2$, $\varphi_{s2}(x_i) = |m_{A2}(x_i) - m_{B2}(x_i)|/2$, $m_{A1}(x_i) = (\mu_A(x_i) + m_A(x_i))/2$, $m_{B1}(x_i) = (\mu_B(x_i) + m_B(x_i))/2$, $m_{A2}(x_i) = (m_A(x_i) + 1 - \nu_A(x_i))/2$, $m_{B2}(x_i) = (m_B(x_i) + 1 - \nu_B(x_i))/2$, $m_A(x_i) = (\mu_A(x_i) + 1 - \nu_A(x_i))/2$, $m_B(x_i) = (\mu_B(x_i) + 1 - \nu_B(x_i))/2;$

$$S_h^p(A,B) = 1 - \sqrt[p]{\frac{\sum_{i=1}^n (\eta_1(i) + \eta_2(i) + \eta_3(i))^p}{3n}}, \quad (5\text{-}27)$$

$\eta_1(i) = \phi_\mu(x_i) + \phi_\nu(x_i)$ (S_e^p) $\eta_1(i) = \varphi_{s1}(x_i) + \varphi_{s2}(x_i)$ (S_s^p), $\eta_2(i) = |\psi_A(x_i) - \psi_B(x_i)|$ (S_{DC}), $\eta_3(i) = max(l_A(i), l_B(i)) - min(l_A(i), l_B(i))$, $l_A(i) = (1 - \mu_A(x_i) - \nu_A(x_i))/2$, $l_B(i) = (1 - \mu_B(x_i) - \nu_B(x_i))/2;$

$$S_{HY}^1(A,B) = 1 - d_H(A,B), \quad (5\text{-}28)$$

$$d_H(A,B)=\frac{1}{n}\sum_{i=1}^{n}max(|\mu_A(x_i)-\mu_B(x_i)|,|\nu_A(x_i)-\nu_B(x_i)|);$$

$$S_{HY}^{2}(A,B)=(e^{-d_H(A,B)}-e^{-1})/(1-e^{-1}); \tag{5-29}$$

$$d_H(A,B)=\frac{1}{n}\sum_{i=1}^{n}max(|\mu_A(x_i)-\mu_B(x_i)|\ |\nu_A(x_i)-\nu_B(x_i)|);$$

$$S_{HY}^{3}(A,B)=(1-d_H(A,B))/(1+d_H(A,B)); \tag{5-30}$$

$$d_H(A,B)=\frac{1}{n}\sum_{i=1}^{n}max(|\mu_A(x_i)-\mu_B(x_i)|\ |\nu_A(x_i)-\nu_B(x_i)|);$$

$$C(A,B)=\frac{1}{n}\sum_{i=1}\frac{\mu_A(x_i)\mu_B(x_i)+\nu_A(x_i)\nu_B(x_i)}{\sqrt{\mu_A^2(x_i)+\nu_A^2(x_i)}\sqrt{\mu_B^2(x_i)+\nu_B^2(x_i)}}. \tag{5-31}$$

6 [31]

13 (5-15) (5.19)-(5.31) 14

5-1

表 5-1 直觉模糊集A 和B之间的相似度

	1	2	3	4	5	6
A	(0.3, 0.3)	(0.3, 0.4)	(1, 0)	(0.5, 0.5)	(0.4, 0.2)	(0.4, 0.2)
B	(0.4, 0.4)	(0.4, 0.3)	(0, 0)	(0.0, 0.0)	(0.5, 0.3)	(0.5, 0.2)
S_C	1.0000	0.9000	0.5000	1.0000	1.0000	0.9500
S_H	0.9000	0.9000	0.5000	0.5000	0.9000	0.9500
S_L	0.9500	0.9000	0.5000	0.7500	0.9500	0.9500
S_O	1.0000	0.9000	0.5000	1.0000	1.0000	0.9500
S_{DC}	0.9500	0.9000	0.5000	0.7500	0.9500	0.9500
S_{HB}	0.9000	0.9000	0.5000	0.5000	0.9000	0.9500
S_e^p	0.9000	0.9000	0.5000	0.5000	0.9000	0.9500
S_s^p	0.9500	0.9000	0.5000	0.7500	0.9500	0.9500
S_h^p	0.9300	0.9330	0.5000	0.6700	0.9300	0.9500
S_{HY}^1	0.9000	0.9000	0.0000	0.5000	0.9000	0.9000
S_{HY}^2	0.8500	0.8500	0.0000	0.3800	0.8500	0.8500
S_{HY}^3	0.8200	0.8200	0.0000	0.3300	0.8200	0.8200
C	1.0000	0.9600	0.0000	0.0000	0.9971	0.9965
E	0.9600	0.9600	0.0000	0.0000	0.9630	0.9796
$S_V^{\alpha=0.2}$	0.9920	0.9600	0.0000	0.0000	0.9902	0.9932
$S_V^{\alpha=0.8}$	0.9680	0.9600	0.0000	0.0000	0.9698	0.9830

5-1

[104] [31]

S_h^p C , $A=<1,0>$ $B=<0,0>$ $S_h^p=0.5$ $A=<0.3,0.3>$ $B=<0.4,0.4>$ $C=1$

5-1

1 $A=\{A_1,A_2,A_3\}$, $C=\{c_1,c_2,c_3\}$, $A_i(i=1,2,3)$ C

$$A_1=\{<c_1,0.3,0.3>,<c_2,0.2,0.2>,<c_3,0.4,0.4>\};$$
$$A_2=\{<c_1,0.8,0.1>,<c_2,1.0,0.0>,<c_3,0.9,0.0>\};$$
$$A_3=\{<c_1,0.4,0.4>,<c_2,0.8,0.8>,<c_3,0.7,0.7>\}.$$

$$AI=\{<c_1,0.2,0.2>,<c_2,0.3,0.3>,<c_3,0.5,0.5>\};$$

$A_i(i=1,2,3)$

5-14 5-15

$$E(AI,A_1)=0.9406;\ \ E(AI,A_2)=0.8887;\ \ E(AI,A_3)=0.8012;$$

$$S_V(AI,A_1)=0.9881;\ \ S_V(AI,A_1)=0.9777;\ \ S_V(AI,A_1)=0.9602.$$

$A_1 \succ A_2 \succ A_3$ A_1

[31] $A_i(i=1,2,3)$

$$C(AI,A_1)=1;\ \ C(AI,A_2)=0.8457;\ \ C(AI,A_3)=1;$$

[31]

[31]

5.4 直觉模糊集相似度的应用

[26],[31],[104]

5.4.1 模式识别案例分析

[26],[31],[104]

C_1, C_2 C_3 ,$X = \{x_1, x_2, x_3\}$

$$C_1 = \{<x_1, 1.0, 0.0>, <x_2, 0.8, 0.0>, <x_3, 0.7, 0.1>\},$$
$$C_2 = \{<x_1, 0.8, 0.1>, <x_2, 1.0, 0.0>, <x_3, 0.9, 0.0>\},$$
$$C_3 = \{<x_1, 0.6, 0.2>, <x_2, 0.8, 0.0>, <x_3, 1.0, 0.0>\}.$$

A X :

$$A = \{<x_1, 0.5, 0.3>, <x_2, 0.6, 0.2>, <x_3, 0.8, 0.1>\}.$$

A C_1, C_2 C_3

A $C_k(k = 1, 2, 3)$

$$k = \max_{1 \le i \le 3}\{S_V(C_i, A)\}. \tag{5-32}$$

$\alpha = 0.8$ 5-15 $C_i(i = 1, 2, 3)$ A

$$S_V(C_1, A) = 0.9256, \quad S_V(C_2, A) = 0.9423, \quad S_V(C_3, A) = 0.9690.$$

5-32 A C_3 [26],[31],[104]

x_1, x_2 x_3 0.5 0.3 0.2 A
$C_k(k = 1, 2, 3)$

$$k = \max_{1 \le i \le 3}\{WS_V(C_i, A)\}. \tag{5-33}$$

$\alpha = 0.8$, 5-15

$$WS_V(C_1, A) = 0.9003, WS_V(C_2, A) = 0.9301, WS_V(C_3, A) = 0.9684,$$

5-33 A C_3 [26],[31],[104]

5.4.2 医学诊断案例分析

[31],[95]

5.4.2

$A = \{A_1($ $)$, $A_2($ $)$, $A_3($ $)$, $A_4($ $)$, $A_5($ $)\}$,
$B = \{b_1($ $)$, $b_2($ $)$, $b_3($ $)$, $b_4($ $)$, $b_5($ $)\}$

$$P(Patient) = \{<b_1, 0.8, 0.1>, <b_2, 0.6, 0.1>, <b_3, 0.2, 0.8>, <b_4, 0.6, 0.1>,$$
$$<b_5, 0.1, 0.6>\}.$$

$A_i(i = 1; 2; 3; 4; 5)$

$$
\begin{aligned}
A_1(Vf) &= f < b_1; 0{:}4; 0{:}0 >; < b_2; 0{:}3; 0{:}5 >; < b_3; 0{:}1; 0{:}7 >; < b_4; 0{:}4; 0{:}3 >; \\
&\quad < b_5; 0{:}1; 0{:}7 > g; \\
A_2(M) &= f < b_1; 0{:}7; 0{:}0 >; < b_2; 0{:}2; 0{:}6 >; < b_3; 0{:}0; 0{:}9 >; < b_4; 0{:}7; 0{:}0 >; \\
&\quad < b_5; 0{:}1; 0{:}8 > g; \\
A_3(T) &= f < b_1; 0{:}3; 0{:}3 >; < b_2; 0{:}6; 0{:}1 >; < b_3; 0{:}2; 0{:}7 >; < b_4; 0{:}2; 0{:}6 >; \\
&\quad < b_5; 0{:}1; 0{:}9 > g; \\
A_4(Sp) &= f < b_1; 0{:}1; 0{:}7 >; < b_2; 0{:}2; 0{:}4 >; < b_3; 0{:}8; 0{:}0 >; < b_4; 0{:}2; 0{:}7 >; \\
&\quad < b_5; 0{:}2; 0{:}7 > g; \\
A_5(Cp) &= f < b_1; 0{:}1; 0{:}8 >; < b_2; 0{:}0; 0{:}8 >; < b_3; 0{:}2; 0{:}8 >; < b_4; 0{:}2; 0{:}8 >; \\
&\quad < b_5; 0{:}8; 0{:}1 > g{:}
\end{aligned}
$$

P $A_1; A_2; A_3; A_4$ A_5
P $A_k(k = 1; 2; 3; 4; 5)$

$$k = \max_{1 \le i \le 5} f S_V(A_i; P) g{:} \tag{5-34}$$

, $® = 0{:}8$, 5-15 :

$$
\begin{aligned}
&S_V(P; A_1) = 0{:}8949; \quad S_V(P; A_2) = 0{:}8807; \quad S_V(P; A_3) = 0{:}8422; \\
&S_V(P; A_4) = 0{:}5012; \quad S_V(P; A_5) = 0{:}4176{:}
\end{aligned}
$$

, 5-34 A_1
[31],[95] $® = 0{:}2$ $® = 0{:}5$

5.4.3 模糊多属性多方案决策案例分析

TOPSIS
Li
[105] Wu Zhang [106]

Wang Li [107]

[107]

[30],[108],[109],[110] -[111]

定义5.8 p $A=\{A_1,A_2,\dots,A_p\}$ n $C=\{C_1,C_2,\dots,C_n\}$ p n i j $v_j^i=<C_j,\mu_v(C_j),\nu_v(C_j)>$ $i=1,2,\dots,m;j=1,2,\dots,n$ $V^i=(v_1^i,v_2^i,\dots v_n^i)$ A_i

: (1) A_1 (2) A_2 (3) A_3 (A_1,A_2,A_3) (1) C_1 (2) C_2 (3) C_3 (4) C_4 $C_j(j=1,2,3,4)$ $A_i(i=1,2,3)$

$$A_1=\{<C_1,0.60,0.40>,<C_2,0.55,0.35>,<C_3,0.40,0.20>,<C_4,0.52,0.45>\};$$
$$A_2=\{<C_1,0.55,0.35>,<C_2,0.65,0.30>,<C_3,0.55,0.04>,<C_4,0.45,0.15>\};$$
$$A_3=\{<C_1,0.60,0.30>,<C_2,0.68,0.30>,<C_3,0.72,0.25>,<C_4,0.65,0.12>\}.$$

[98]

$$A^+=\{<C_j,\mu_{A^+}(C_j),\nu_{A^+}(C_j)>|C_j\in C\};\tag{5-35}$$
$$A^-=\{<C_j,\mu_{A^-}(C_j),\nu_{A^-}(C_j)>|C_j\in C\};\tag{5-36}$$

$$\mu_{A^+}(C_j)=\max_i\{\mu_{A_i}(C_j)\};\quad \nu_{A^+}(C_j)=\min_i\{\nu_{A_i}(C_j)\};$$
$$\mu_{A^-}(C_j)=\min_i\{\mu_{A_i}(C_j)\};\quad \nu_{A^-}(C_j)=\max_i\{\nu_{A_i}(C_j)\}.$$

(5-35) 5-36 $A_i(i=1,2,3)$ A^+ A^-

$$A^+=\{<C_1,0.60,0.30>,<C_2,0.68,0.30>,<C_3,0.72,0.04>,<C_4,0.65,0.12>\};$$

$$A^-=\{<C_1,0.55,0.40>,<C_2,0.55,0.30>,<C_3,0.40,0.25>,<C_4,0.45,0.45>\}.$$

(5-14) $A_i(i=1,2,3)$ A^+ E-

$$S_E(A^+,A_1)=0.9134;\quad S_E(A^+,A_2)=0.9741;\quad S_E(A^+,A_3)=0.9900;$$

5-14 $A_i(i=1,2,3)$ A^- E-

$$S_E(A^-,A_1)=0.9957,\quad S_E(A^-,A_2)=0.9292,\quad S_E(A^-,A_3)=0.9160.$$

5-35 5-36

R A_i A^+ A^- R_i

$$R_i=\frac{S_E(A^+,A_i)}{S_E(A^-,A_i)},\ i=1,2,\dots,n. \qquad (5\text{-}37)$$

(5-37)

$$R_1=0.9174,\quad R_2=1.0484,\quad R_3=1.0807.$$

$A_3\succ A_2\succ A_1$,

A_3

$C_j(j=1,2,3,4)$

$W=(0.17,0.3,0.4,0.13)$

(5-18)

$$WS_E(A^+,A_1)=0.9032,\quad WS_E(A^+,A_2)=0.9767,\quad WS_E(A^+,A_3)=0.9840,$$
$$WS_E(A^-,A_1)=0.9956,\quad WS_E(A^-,A_2)=0.9270,\quad WS_E(A^-,A_3)=0.9183.$$

(5-37)

$$WR_1=0.9072,\quad WR_2=1.0536,\quad WR_3=1.0715.$$

$A_3\succ A_2\succ A_1$ A_3

5.5 基于直觉模糊集得分矢量间相似度的多属性群决策方法

5.5.1 直觉模糊集的得分函数

$\alpha=\{<x_j,\mu_\alpha(x_j),\nu_\alpha(x_j)>|x_j\in X\}$ $\beta=\{<x_j,\mu_\beta(x_j),\nu_\beta(x_j)>|x_j\in X\}$ $X=\{x_1,x_2,\dots,x_n\}$

$$\alpha+\beta=\{<x_j,\mu_\alpha(x_j)+\mu_\beta(x_j),\nu_\alpha(x_j)+\nu_\beta(x_j)>|x_j\in X\};$$
$$\alpha\beta=\{<x_j,\mu_\alpha(x_j)\mu_\beta(x_j),\nu_\alpha(x_j)\nu_\beta(x_j)>|x_j\in X\};$$
$$\lambda\alpha=\{<x_j,\lambda\mu_\alpha(x_j),\lambda\nu_\alpha(x_j)>|x_j\in X\}.$$

$$\alpha \subseteq \beta \; iff \; \{\mu_\alpha(x) \le \mu_\beta(x), \nu_\alpha(x) \ge \nu_\beta(x) \mid \forall x \in X\}$$
$$\alpha = \beta \; iff \; \{\mu_\alpha(x) = \mu_\beta(x), \nu_\alpha(x) = \nu_\beta(x) \mid \forall x \in X\}$$

[112],[113],[114]

Chen Tan

[115]

定义5.9[115] $\alpha = (\mu, \nu)$ X X

$$D(\alpha) = \mu - \nu \tag{5-38}$$

α $D(\alpha)$ $D(\alpha) \in [-1, 1]$ α

α

Hong Choi [116]

定义5.10[116] $\alpha = (\mu, \nu)$ X X

$$F(\alpha) = \mu + \nu \tag{5-39}$$

α $F(\alpha)$ $F(\alpha) \in [0, 2]$

α α

5.5.2 直觉模糊集的得分矢量之间的相似度

定义5.10 p $A = \{A_1, A_2, ..., A_p\}$ n $C = \{C_1, C_2, ..., C_n\}$

$$A_i = \{(C_1, < \mu_{i1}, \nu_{i1} >), (C_2, < \mu_{i2}, \nu_{i2} >), ..., (C_m, < \mu_{im}, \nu_{im} >)\};$$

μ_{ij} A_i C_j ν_{ij} A_i C_j 5-38 A_i

$$D_i = (d_{i1}, d_{i2}, ..., d_{im}) = (\mu_{i1} - \nu_{i1}, \mu_{i2} - \nu_{i2}, ..., \mu_{im} - \nu_{im}). \tag{5-40}$$

5-40

[redacted] $; 2; \dots; n;$ (5-47) $S_E^{IFS}(A_P; A_i)$

i 5-40

[redacted] $\frac{d_{i1} - D_i^1}{D_i^2 - D_i^1}, \frac{d_{i2} - D_i^1}{D_i^2 - D_i^1}, \dots, \frac{d_{im} - D_i^1}{D_i^2 - D_i^1});$ (5-41)

[redacted] $D_i^2 = \max\{d_{i1}, d_{i2}, \dots, d_{im}\}.$

[redacted] E- C-

[redacted] A_i A_j

S_J^{IF} [redacted] $- \tilde{D}_i \tilde{D}_j} = \frac{\sum_{k=1}^{m} \tilde{d}_{ik} \tilde{d}_{jk}}{\sum_{k=1}^{m} \tilde{d}_{ik}^2 + \sum_{k=1}^{m} \tilde{d}_{jk}^2 - \sum_{k=1}^{m} \tilde{d}_{ik} \tilde{d}_{jk}};$ (5-42)

[redacted] $\frac{2\tilde{D}_i \tilde{D}_j}{\|\tilde{D}_i\|_2^2 + \|\tilde{D}_j\|_2^2} = \frac{2\sum_{k=1}^{m} \tilde{d}_{ik} \tilde{d}_{jk}}{\sum_{k=1}^{m} \tilde{d}_{ik}^2 + \sum_{k=1}^{m} \tilde{d}_{jk}^2};$ (5-43)

[redacted] $\frac{\tilde{D}_i \tilde{D}_j}{\|\tilde{D}_i\|_2 \|\tilde{D}_j\|_2} = \frac{\sum_{k=1}^{m} \tilde{d}_{ik} \tilde{d}_{jk}}{\sqrt{\sum_{k=1}^{m} \tilde{d}_{ik}^2}\sqrt{\sum_{k=1}^{m} \tilde{d}_{jk}^2}}.$ (5-44)

[redacted] $\mu_{A^+}(C_j) \rangle \mid C_j \in C\}; \ i = 1, 2, \dots, p;$ (5-45)

[redacted] $; \mu_{A^-}(C_j) \rangle \mid C_j \in C\}; \ i = 1, 2, \dots, p.$ (5-46)

[redacted] $(C_j)\}; \ i = 1, 2, \dots, p; \ j = 1, 2, \dots, n;$

[redacted] $(C_j)\}; \ i = 1, 2, \dots, p; \ j = 1, 2, \dots, n;$

[redacted] $(C_j)\}; \ i = 1, 2, \dots, p; \ j = 1, 2, \dots, n;$

[redacted] $_i(C_j)\}; \ i = 1, 2, \dots, p; \ j = 1, 2, \dots, n.$

[redacted] A_i R_i

[redacted] $S(A_P; A_i)$ / $\frac{1}{2}(S_J$ [redacted] $(A_P; A_i) + S_J^{IFS}(A_N; A_i))$; $i = 1, 2, \dots, n;$ (5-47)

$ER_i = \frac{S_E^{IFS}(A_P; A_i)}{\dots}$

5.5.3 基于得分矢量相似度的模糊多属性大群决策方法

$A = \{A_1, A_2, \ldots, A_p\}$ $C = \{C_1, C_2, \ldots, C_n\}$

$G = \{G_1, G_2, \ldots, G_m\}$ m

G_j A_i

$$v_i^j = \{<C_1, \mu^j_{A_i}(C_1), \nu^j_{A_i}(C_1)>, <C_2, \mu^j_{A_i}(C_2), \nu^j_{A_i}(C_2)>, \ldots, <C_n, \mu^j_{A_i}(C_n), \nu^j_{A_i}(C_n)>\};$$

$i = 1, 2, \ldots, p; j = 1, 2, \ldots, m$ $\mu_{A_i}(C_k)$ A_i C_k $\nu_{A_i}(C_k)$ A_i C_k $V^j = (v_1^j, v_2^j, \ldots, v_n^j)$ G_j

步骤1 5-45 - 5-46 A_P A_N

步骤2 5-38 A_i n T_i $(i = 1, 2, \ldots, n)$

$$T_i = \begin{pmatrix} t_{11} & t_{12} & \cdots & t_{1n} \\ t_{21} & t_{22} & \cdots & t_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ t_{p1} & t_{p2} & \cdots & t_{pn} \end{pmatrix} =$$

$$\begin{pmatrix} \mu^1_{A_1}(C_1) - \nu^1_{A_i}(C_1) & \mu^1_{A_1}(C_2) - \nu^1_{A_1}(C_2) & \cdots & \mu^1_{A_1}(C_n) - \nu^1_{A_1}(C_n) \\ \mu^2_{A_2}(C_1) - \nu^2_{A_2}(C_1) & \mu^2_{A_i}(C_2) - \nu^2_{A_2}(C_2) & \cdots & \mu^2_{A_2}(C_n) - \nu^2_{A_2}(C_n) \\ \vdots & \vdots & \vdots & \vdots \\ \mu^l_{A_p}(C_1) - \nu^l_{A_p}(C_1) & \mu^l_{A_p}(C_2) - \nu^l_{A_p}(C_2) & \cdots & \mu^l_{A_p}(C_n) - \nu^l_{A_p}(C_n) \end{pmatrix}.$$

步骤3 T_i

$$\tilde{t}(i, j) = \frac{t(i, j) - T_1(i)}{T_2(i) - T_1(i)}, \quad (5\text{-}50)$$

$T_1(i) = \min\{t_{i1}, t_{i2}, \ldots, t_{in}\}$ $T_2(i) = \max\{t_{i1}, t_{i2}, \ldots, t_{in}\}$ $T_1(i)$ T_i i $T_2(i)$ i $i = 1, 2, \ldots, l$

$\tilde{T}_i = (\tilde{t}(i, j))_{p \times n}$

步骤4 $W=(w_1,w_2,\ldots,w_n)$ p A_i

$$U_i=W'\tilde{R}_i=(w_1,w_2,\ldots,w_m)\begin{pmatrix}\tilde{r}_{11} & \tilde{r}_{12} & \cdots & \tilde{r}_{1n}\\ \tilde{r}_{21} & \tilde{r}_{22} & \cdots & \tilde{r}_{2n}\\ \vdots & \vdots & \vdots & \vdots\\ \tilde{r}_{m1} & \tilde{r}_{m2} & \cdots & \tilde{r}_{mn}\end{pmatrix}$$

$$=(u_i^1,u_i^2,\ldots,u_i^n). \tag{5-51}$$

步骤5 5-40 - 5-41

D_P D_N

步骤6 (5-42)-(5-44) (5-47)-(5-49) $A_i\ (i=1,2,\ldots,n)$

$$o_J=\max_{1\le i\le n}\{JR_i\},\ o_E=\max_{1\le i\le n}\{ER_i\},\ o_C=\max_{1\le i\le n}\{CR_i\}. \tag{5-52}$$

5.5.4 模糊偏好多属性大群体应急决策案例分析

PWR5 ,

40km

, 5 : (1) A_1 (2) A_2

25km 11. 7 (3)

A_3 11 km 11 25 km

1 1 (4) A_4 2,

12 70

(5) A_5 20km

7.4 80

$A=\{A_1,A_2,A_3,A_4,A_5\}$

5 (1) C_1

(2) C_2 (3) C_3 (4) C_4

5-2

步骤1 5-45 - 5-46

表 5-2 5个专家对5个方案的模糊偏好矢量表

A	M	C_1	C_2	C_3	C_4
A_1	1	$<0:05;0:09>$	$<0:29;0:30>$	$<0:08;0:46>$	$<0:13;0:05>$
	2	$<0:27;0:36>$	$<0:07;0:29>$	$<0:46;0:30>$	$<0:06;0:33>$
	3	$<0:31;0:05>$	$<0:41;0:07>$	$<0:32;0:35>$	$<0:18;0:22>$
	4	$<0:24;0:44>$	$<0:25;0:27>$	$<0:10;0:22>$	$<0:49;0:27>$
	5	$<0:17;0:46>$	$<0:33;0:25>$	$<0:34;0:47>$	$<0:41;0:36>$
A_2	1	$<0:41;0:16>$	$<0:36;0:17>$	$<0:40;0:26>$	$<0:44;0:46>$
	2	$<0:37;0:20>$	$<0:03;0:44>$	$<0:48;0:28>$	$<0:20;0:41>$
	3	$<0:81;0:12>$	$<0:63;0:27>$	$<0:76;0:03>$	$<0:70;0:27>$
	4	$<0:09;0:69>$	$<0:39;0:11>$	$<0:03;0:38>$	$<0:18;0:44>$
	5	$<0:70;0:27>$	$<0:65;0:11>$	$<0:67;0:12>$	$<0:49;0:34>$
A_3	1	$<0:22;0:25>$	$<0:54;0:14>$	$<0:61;0:13>$	$<0:25;0:25>$
	2	$<0:24;0:35>$	$<0:25;0:47>$	$<0:38;0:56>$	$<0:01;0:33>$
	3	$<0:16;0:79>$	$<0:31;0:52>$	$<0:16;0:60>$	$<0:26;0:65>$
	4	$<0:45;0:08>$	$<0:15;0:82>$	$<0:07;0:44>$	$<0:06;0:96>$
	5	$<0:04;0:77>$	$<0:08;0:39>$	$<0:18;0:26>$	$<0:14;0:13>$
A_4	1	$<0:54;0:14>$	$<0:35;0:51>$	$<0:40;0:07>$	$<0:23;0:12>$
	2	$<0:18;0:24>$	$<0:41;0:04>$	$<0:49;0:48>$	$<0:36;0:11>$
	3	$<0:24;0:40>$	$<0:09;0:13>$	$<0:57;0:05>$	$<0:23;0:35>$
	4	$<0:82;0:01>$	$<0:03;0:73>$	$<0:64;0:35>$	$<0:54;0:29>$
	5	$<0:74;0:18>$	$<0:68;0:18>$	$<0:36;0:62>$	$<0:78;0:08>$
A_5	1	$<0:92;0:00>$	$<0:48;0:43>$	$<0:44;0:30>$	$<0:50;0:41>$
	2	$<0:81;0:09>$	$<0:64;0:31>$	$<0:35;0:63>$	$<0:20;0:30>$
	3	$<0:47;0:23>$	$<0:74;0:19>$	$<0:22;0:17>$	$<0:22;0:43>$
	4	$<0:03;0:92>$	$<0:43;0:18>$	$<0:43;0:11>$	$<0:25;0:40>$
	5	$<0:59;0:26>$	$<0:22;0:11>$	$<0:29;0:31>$	$<0:42;0:50>$

$$
\begin{aligned}
A_P = {} & f < C_1; 0:9294; 0:0057 >; < C_2; 0:7443; 0:0078 >; < C_3; 0:7690; 0:0598 >; \\
& < C_4; 0:7802; 0:0504 > g; \\
A_N = {} & f < C_1; 0:0046; 0:9234 >; < C_2; 0:0430; 0:8358 >; < C_3; 0:0782; 0:6390 >; \\
& < C_4; 0:0119; 0:9616 > g:
\end{aligned}
$$

步骤2 (5-38)

$$T_1 = \begin{pmatrix} -0.0372 & -0.0349 & -0.3838 & 0.0853 \\ -0.1076 & -0.2081 & 0.1074 & -0.2745 \\ 0.2637 & 0.4093 & 0.0393 & -0.0471 \\ -0.2044 & -0.0198 & -0.1259 & 0.2216 \\ -0.2964 & 0.0723 & -0.1257 & 0.0524 \end{pmatrix};$$

$$T_2 = \begin{pmatrix} 0.2564 & 0.1867 & 0.1438 & -0.0225 \\ 0.1713 & -0.4084 & 0.2017 & -0.2159 \\ 0.6877 & 0.3539 & 0.7333 & 0.4291 \\ -0.5977 & 0.2210 & -0.3472 & -0.2587 \\ 0.4334 & 0.5361 & 0.5171 & 0.1580 \end{pmatrix};$$

$$T_3 = \begin{pmatrix} -0.0313 & 0.3979 & 0.4774 & 0.0032 \\ -0.1065 & -0.2222 & -0.1874 & -0.3252 \\ -0.6321 & -0.2173 & -0.4364 & -0.3911 \\ 0.3667 & -0.6734 & -0.3645 & -0.9552 \\ -0.7703 & -0.3154 & -0.0820 & 0.0094 \end{pmatrix};$$

$$T_4 = \begin{pmatrix} 0.4049 & -0.1622 & 0.3258 & 0.1166 \\ -0.0561 & 0.3676 & 0.0016 & 0.2580 \\ -0.1622 & -0.0355 & 0.5154 & -0.1184 \\ 0.8058 & -0.6887 & 0.2968 & 0.2507 \\ 0.5557 & 0.5033 & -0.2571 & 0.6991 \end{pmatrix};$$

$$T_5 = \begin{pmatrix} 0.9237 & 0.0509 & 0.1405 & 0.0977 \\ 0.7228 & 0.3257 & -0.2883 & -0.0935 \\ 0.2404 & 0.5495 & 0.0552 & -0.2080 \\ -0.8923 & 0.2454 & 0.3278 & -0.1506 \\ 0.3327 & 0.1043 & -0.0221 & -0.0837 \end{pmatrix}:$$

步骤3 5-50

$$\tilde{T}_1 = \begin{pmatrix} 0.7389 & 0.7438 & 0.0000 & 1.0000 \\ 0.4370 & 0.1739 & 1.0000 & 0.0000 \\ 0.6810 & 1.0000 & 0.1893 & 0.0000 \\ 0.0000 & 0.4333 & 0.1843 & 1.0000 \\ 0.0000 & 1.0000 & 0.4630 & 0.9460 \end{pmatrix};$$

$$\tilde{F}_2 = \begin{pmatrix} 1.0000 & 0.7501 & 0.5963 & 0.0000 \\ 0.9502 & 0.0000 & 1.0000 & 0.3155 \\ 0.8798 & 0.0000 & 1.0000 & 0.1982 \\ 0.0000 & 1.0000 & 0.3060 & 0.4141 \\ 0.7284 & 1.0000 & 0.9497 & 0.0000 \end{pmatrix};$$

$$\tilde{F}_3 = \begin{pmatrix} 0.0000 & 0.8437 & 1.0000 & 0.0678 \\ 1.0000 & 0.4710 & 0.6301 & 0.0000 \\ 0.0000 & 1.0000 & 0.4718 & 0.5810 \\ 1.0000 & 0.2132 & 0.4469 & 0.0000 \\ 0.0000 & 0.5834 & 0.8828 & 1.0000 \end{pmatrix};$$

$$\tilde{F}_4 = \begin{pmatrix} 1.0000 & 0.0000 & 0.8605 & 0.4916 \\ 0.0000 & 1.0000 & 0.1362 & 0.7413 \\ 0.0000 & 0.1870 & 1.0000 & 0.0646 \\ 1.0000 & 0.0000 & 0.6594 & 0.6286 \\ 0.8500 & 0.7952 & 0.0000 & 1.0000 \end{pmatrix};$$

$$\tilde{F}_5 = \begin{pmatrix} 1.0000 & 0.0000 & 0.1027 & 0.0536 \\ 1.0000 & 0.6073 & 0.0000 & 0.1927 \\ 0.5919 & 1.0000 & 0.3475 & 0.0000 \\ 0.0000 & 0.9325 & 1.0000 & 0.6079 \\ 1.0000 & 0.4515 & 0.1479 & 0.0000 \end{pmatrix}.$$

步骤4 $W' = (0.1, 0.3, 0.2, 0.3, 0.1)$

5-51

$U_1 = ($[illegible]$, 0.5999,$ [illegible] 得 [illegible] $U_2 = ($[illegible]$)$ [illegible]

$U_3 = (0.7000, 0.5693, 0.6504, 0.2230); \quad U_4 = (0.5850, 0.4169, 0.5907, 0.6359);$

$U_5 = (0.6184, 0.8003, 0.4946, 0.3063).$ V V

¤

表 5-3 基于直觉模糊集得分矢量间的三种相似度的决策结果

	S_J^{IFSP}	S_J^{IFSN}	JR	S_E^{IFSP}	S_E^{IFSN}	ER	S_C^{IFSP}	S_C^{IFSN}	CR
A_1	0.298	0.490	0.117	0:229	0:329	0:064	0.459	0.659	0.257
A_2	0.428	0.689	0.239	0.300	0:408	0:106	0.608	0.820	0.434
A_3	0.522	0.615	**0.297**	0:343	0:381	**0.124**	0.691	0.762	**0.502**
A_4	0.438	0.493	0.204	0:340	0.330	0:096	0.612	0.661	0.390
A_5	0.457	0.520	0.224	0:314	0:342	0:103	0.634	0.686	0.419
Best			A_3			A_3			A_3

5-3 A_3

5-1 5-2 5-3

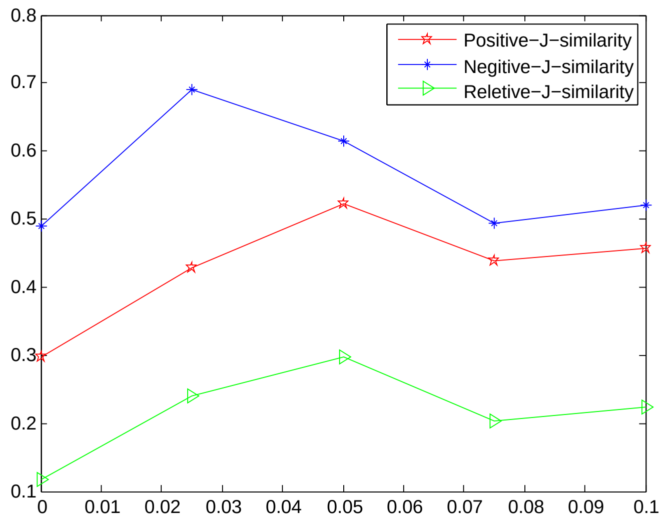

图 5-1 J-相对相似度曲线

A_i $(i =$
$1; 2; :::; 5)$ A_i $(i = 1; 2; :::; 5)$

5.5.5 与其他方法比较

1

[98] [117]

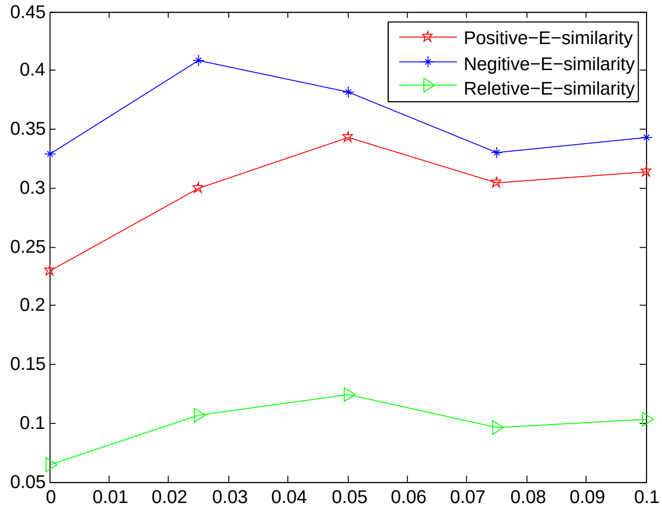

图 5-2 E-相对相似度曲线

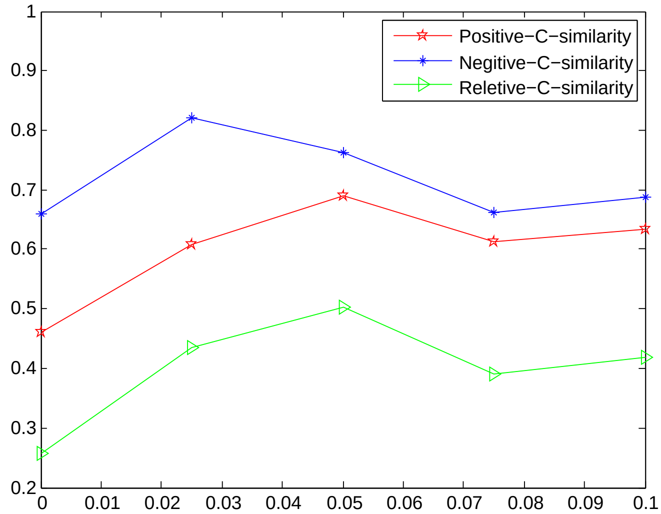

图 5-3 C-相对相似度曲线

$A_i\ (i = 1; 2{:}{:}{:}; 5)$

1 C_1 2 C_2 3 C_3

$A_i\ (i = 1; 2{:}{:}{:}; 5)$

$$A_1 = f< C_1; 0{:}2; 0{:}4 >; < C_2; 0{:}7; 0{:}1 >; < C_3; 0{:}6; 0{:}3 > g;$$

$$A_2 = f< C_1; 0{:}4; 0{:}2 >; < C_2; 0{:}5; 0{:}2 >; < C_3; 0{:}8; 0{:}1 > g;$$

$$A_3 = f< C_1; 0{:}5; 0{:}4 >; < C_2; 0{:}6; 0{:}2 >; < C_3; 0{:}9; 0{:}0 > g;$$

$$A_4 = f< C_1; 0{:}3; 0{:}5 >; < C_2; 0{:}8; 0{:}1 >; < C_3; 0{:}7; 0{:}2 > g;$$

$$A_5 = f< C_1; 0{:}8; 0{:}2 >; < C_2; 0{:}7; 0{:}0 >; < C_3; 0{:}1; 0{:}6 > g{:}$$

5-45 5-46 $A_i\ (i = 1; 2{:}{:}{:}; 5)$

A_P A_N

$$A_P = f < C_1; 0{:}8; 0{:}2 >; < C_2; 0{:}8; 0{:}0 >; < C_3; 0{:}9; 0{:}0 > g;$$
$$A_N = f < C_1; 0{:}2; 0{:}5 >; < C_2; 0{:}5; 0{:}2 >; < C_3; 0{:}1; 0{:}6 > g{:}$$

A_i $(i = 1; 2{:}{:}{:}; 5)$ A_P A_N

$$D_1 = (0{:}0; 1{:}0; 0{:}6);\ D_2 = (0{:}0; 0{:}2; 1{:}0);\ D_3 = (0{:}0; 0{:}4; 1{:}0);$$
$$D_4 = (0{:}0; 1{:}0; 0{:}8);\ D_5 = (0{:}9; 1{:}0; 0{:}0);\ D_P = (0{:}0; 0{:}7; 1{:}0);$$
$$D_N = (0{:}3; 1{:}0; 0{:}0){:}$$

(5-42)-(5-44) (5-47)-(5-49) A_i $(i = 1; 2; {:}{:}{:}; 5)$ J-
E- C-

$$JR_1 = 0{:}5566;\ JR_2 = \mathbf{0.8843};\ JR_3 = 0{:}8161;\ JR_4 = 0{:}6066;\ JR_5 = 0{:}2732;$$
$$ER_1 = 0{:}5320;\ ER_2 = \mathbf{0.8240};\ ER_3 = 0{:}7368;\ ER_4 = 0{:}5617;\ ER_5 = 0{:}3382;$$
$$CR_1 = 0{:}5301;\ CR_2 = \mathbf{0.8267};\ CR_3 = 0{:}7388;\ CR_4 = 0{:}5566;\ CR_5 = 0{:}3308{:}$$

A_2

5.6 本章小结

E-

[31]

6 基于模糊数偏好的多属性大群体决策方法

[50]

[51] [52] [53]

[54] [34],[55] [41] [38]

6.1 模糊数及其拓展形式

6.1.1 引言

[118]

Zadeh 1965 [20]

Atanassov

[25]

[39]

[39] Liu [37]

Bellman Zadeh, 1970

[43]

TOPSIS

Hamming

[119] Hausdor® [29] Euclidean [98]

Cosine [31]

6.1.2 模糊数、直觉模糊数和区间直觉模糊数

定义6.1 ® *R* [118]

8
<
:

¹®(x) =

[122] X x X

$\iota_{®}(x)$ $o_{®}(x)$

6.2.3

6.2.1 三角模糊数及三角模糊数间的相似度

Laarhoven Pedrycz [130] ®

定义6.4[130] ® $(a_1; a_2; a_3)$

® $¹_®(x)$

$$¹_®(x) = \begin{cases} 0; & x < a_1; \\ \frac{x-a_1}{a_2-a_1}; & a_1 · x · a_2; \\ \frac{x-a_3}{a_2-a_3}; & a_2 · x · a_3; \\ 0; & a_3 < x: \end{cases} \quad (6\text{-}5)$$

$0 · a_1 · a_2 · a_3$ a_1 a_3 ® a_2

6-1

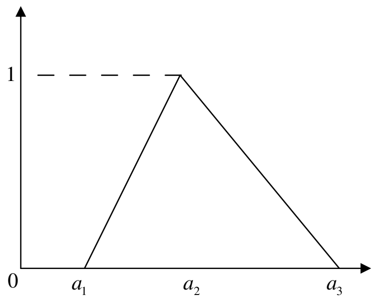

图 6-1 三角模糊数的隶属函数图形

$® = (a_1; a_2; a_3)$ $¯ = (b_1; b_2; b_3)$ °

$$®©¯ = (a_1 + b_1; a_2 + b_2; a_3 + b_3);$$

$$® - ¯ = (a_1 b_1; a_2 b_2; a_3 b_3);$$

$$° - ® = (°a_1; °a_2; °a_3);$$

$$\frac{1}{®} = (\frac{1}{a_1}; \frac{1}{a_2}; \frac{1}{a_3}):$$

®

6.5

定义6.5 $® = (a_1; a_2; a_3)$ $^- = (b_1; b_2; b_3)$
$0 \cdot a_1 \cdot a_2 \cdot a_3$, $0 \cdot b_1 \cdot b_2 \cdot b_3$

$$S^J(®; ^-) = \frac{\sum_{i=1} a_i b_i}{a_i \; i \; \sum}$$

m p n $G = \{G_1, G_2, \ldots, G_m\}$
k G_k i A_i

$$v_i^k = \{< C_1, (a_{i11}^k, a_{i12}^k, a_{i13}^k) >, < C_2, (a_{i21}^k, a_{i22}^k, a_{i23}^k) >, \ldots, < C_n, (a_{in1}^k, a_{in2}^k, a_{in3}^k) >\},$$

$i = 1, 2, \ldots, p; k = 1, 2, \ldots, m.$

m $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_m)$ A_i

$$U_i = \{< C_1, (\sum_{k=1}^{m} \lambda_k a_{i11}^k, \sum_{k=1}^{m} \lambda_k a_{i12}^k, \sum_{k=1}^{m} \lambda_k a_{i13}^k) >, < C_2, (\sum_{k=1}^{m} \lambda_k a_{i21}^k, \sum_{k=1}^{m} \lambda_k a_{i22}^k, \sum_{k=1}^{m} \lambda_k a_{i23}^k) >, \ldots, < C_n, (\sum_{k=1}^{m} \lambda_k a_{in1}^k, \sum_{k=1}^{m} \lambda_k a_{in2}^k, \sum_{k=1}^{m} \lambda_k a_{in3}^k) >\}. \tag{6-12}$$

Heilpern [142]

C_j $(j = 1, 2, \ldots, n)$
C_j
$w_j = (a_{j1}, a_{j2}, a_{j3})$ $(j = 1, 2, \ldots, n)$ w_j $(j = 1, 2, \ldots, n)$

$$EV(w_j) = \frac{a_{j1} + a_{j2} + a_{j3}}{3}. \tag{6-13}$$

(6-13) w_j $(j = 1, 2, \ldots, n)$

$$\hat{w}_j = \frac{EV(w_j)}{\sum_{i=1}^{n} EV(w_j)}. \tag{6-14}$$

[143]

A_i A_{IP}

$$S^J(A_i;A_{IP}) = \sum_{j=1} w_j \frac{\sum_{l=1} a_{ijl}b_{IPjl}}{\sum_{l=1} a_{ijl}^2 + \sum_{l=1} b_{IPjl}^2 - \sum_{l=1} a_{ijl}b_{IPjl}}; \tag{6-15}$$

$$S^E(A_i;A_{IP}) = \sum_{j=1} w_j \frac{2\sum_{l=1} a_{ijl}b_{IPjl}}{\sum_{l=1} a_{ijl}^2 + \sum_{l=1} b_{IPjl}^2}; \tag{6-16}$$

$$S^C(A_i;A_{IP}) = \sum_{j=1} w_j \frac{\sum_{l=1} a_{ijl}b_{IPjl}}{\sqrt{\sum_{l=1} a_{ijl}^2}\sqrt{\sum_{l=1} b_{IPjl}^2}}. \tag{6-17}$$

步骤1 A_i $(i = 1, 2, \ldots, p)$ C_j $(j = 1, 2, \ldots, n)$

步骤2 6-12 A_i $(i = 1, 2, \ldots, p)$

步骤3 6-13 6-14 C_j $(j = 1, 2, \ldots, n)$ w_j

步骤4 (6-15) - (6-17) A_i $(i = 1, 2, \ldots, p)$ J- E- C-

步骤5 p

6.2.3 案例分析

A_1 A_2 A_3 A_4 5 5 (1) (C_1) (2) (C_2) (3) (C_3) (4) (C_4) (5) (C_5) f g 5 5-1

表 6-1 语言值与三角模糊数的对照表

(AP)	(0.0,0.0,0.1)
(VP)	(0.0,0.1,0.2)
(P)	(0.1,0.2,0.3)
(MP)	(0.3,0.4,0.5)
(M)	(0.4,0.5,0.6)
(MD)	(0.5,0.6,0.7)
(G)	(0.7,0.8,0.9)
(VG)	(0.8,0.9,1.0)
(AG)	(0.9,1.0,1.0)

步骤1 5

6-2

步骤2 6-12) $A_i\ (i = 1; 2; 3; 4)$

$$U_1 = f < C_1; (0{:}13; 0{:}17; 0{:}22) >; < C_2; (0{:}36; 0{:}46; 0{:}56) >; < C_3; (0{:}39; 0{:}54; 0{:}69) >;$$
$$< C_4; (0{:}42; 0{:}50; 0{:}60) >; < C_5; (0{:}52; 0{:}60; 0{:}60) > g;$$

$$U_2 = f < C_1; (0{:}22; 0{:}27; 0{:}32) >; < C_2; (0{:}54; 0{:}64; 0{:}74) >; < C_3; (0{:}60; 0{:}75; 0{:}90) >;$$
$$< C_4; (0{:}44; 0{:}54; 0{:}62) >; < C_5; (0{:}18; 0{:}26; 0{:}26) > g;$$

$$U_3 = f < C_1; (0{:}18; 0{:}23; 0{:}27) >; < C_2; (0{:}28; 0{:}36; 0{:}46) >; < C_3; (0{:}69; 0{:}84; 0{:}99) >;$$
$$< C_4; (0{:}16; 0{:}24; 0{:}34) >; < C_5; (0{:}48; 0{:}58; 0{:}58) > g;$$

$$U_4 = f < C_1; (0{:}21; 0{:}26; 0{:}31) >; < C_2; (0{:}34; 0{:}42; 0{:}52) >; < C_3; (0{:}57; 0{:}72; 0{:}87) >;$$
$$< C_4; (0{:}28; 0{:}38; 0{:}48) >; < C_5; (0{:}46; 0{:}56; 0{:}56) > g{:}$$

步骤3 C_i

$$w_1 = < C_1; (0{:}3; 0{:}4; 0{:}5) >; w_2 = < C_2; (0{:}5; 0{:}6; 0{:}7) >; w_3 = < C_3; (0{:}1; 0{:}2; 0{:}3) >;$$
$$w_4 = < C_4; (0{:}9; 1{:}0; 1{:}0) >; w_5 = < C_5; (0{:}4; 0{:}5; 0{:}7) > {:}$$

(6-13) 6-14

5 $W = (0.1481, 0.2222, 0.0741, 0.3580, 0.1975)$

表 6-2 5个专家的三角模糊偏好矢量表

A	M	C_1	C_2	C_3	C_4	C_5
A_1	1	(0:0; 0:0; 0:1)	(0:4; 0:5; 0:6)	(0:5; 0:6; 0:7)	(0:8; 0:9; 1:0)	(0:5; 0:6; 0:7)
	2	(0:1; 0:2; 0:3)	(0:3; 0:4; 0:5)	(0:4; 0:5; 0:6)	(0:5; 0:6; 0:7)	(0.9,1.0,1.0)
	3	(0:4; 0:5; 0:6)	(0:4; 0:5; 0:6)	(0:1; 0:2; 0:3)	(0:3; 0:4; 0:5)	(0:0; 0:0; 0:1)
	4	(0:3; 0:4; 0:5)	(0:1; 0:2; 0:3)	(0:0; 0:1; 0:2)	(0:5; 0:6; 0:7)	(0:7; 0:8; 0:9)
	5	(0:5; 0:6; 0:7)	(0:7; 0:8; 0:9)	(0:2; 0:3; 0:4)	(0:0; 0:0; 0:1)	(0:5; 0:6; 0:7)
A_2	1	(0:4; 0:5; 0:6)	(0:8; 0:9; 1:0)	(0:5; 0:6; 0:7)	(0:4; 0:5; 0:6)	(0:0; 0:1; 0:2)
	2	(0:1; 0:2; 0:3)	(0:4; 0:5; 0:6)	(0:5; 0:6; 0:7)	(0:8; 0:9; 1:0)	(0:1; 0:2; 0:3)
	3	(0:7; 0:8; 0:9)	(0:5; 0:6; 0:7)	(0:3; 0:4; 0:5)	(0:1; 0:2; 0:3)	(0:0; 0:1; 0:2)
	4	(0:5; 0:6; 0:7)	(0:7; 0:8; 0:9)	(0:3; 0:4; 0:5)	(0:9; 1:0; 1:0)	(0:0; 0:0; 0:1)
	5	(0:5; 0:6; 0:7)	(0:3; 0:4; 0:5)	(0:4; 0:5; 0:6)	(0:0; 0:1; 0:2)	(0:8; 0:9; 1:0)
A_3	1	(0:3; 0:4; 0:5)	(0:1; 0:2; 0:3)	(0:7; 0:8; 0:9)	(0:4; 0:5; 0:6)	(0:5; 0:6; 0:7)
	2	(0:1; 0:2; 0:3)	(0:5; 0:6; 0:7)	(0:3; 0:4; 0:5)	(0:0; 0:0; 0:1)	(0:9; 1:0; 1:0)
	3	(0:9; 1:0; 1:0)	(0:4; 0:5; 0:6)	(0:5; 0:6; 0:7)	(0:0; 0:1; 0:2)	(0:3; 0:4; 0:5)
	4	(0:1; 0:2; 0:3)	(0:0; 0:0; 0:1)	(0:5; 0:6; 0:7)	(0:4; 0:5; 0:6)	(0:0; 0:1; 0:2)
	5	(0.4,0.5,0.6)	(0.4,0.5,0.6)	(0.3,0.4,0.5)	(0.0,0.1,0.2)	(0.7,0.8,0.9)
A_4	1	(0.5,0.6,0.7)	(0.5,0.6,0.7)	(0.4,0.5,0.6)	(0.3,0.4,0.5)	(0.8,0.9,1.0)
	2	(0.4,0.5,0.6)	(0.3,0.4,0.5)	(0.5,0.6,0.7)	(0.0,0.1,0.2)	(0.4,0.5,0.6)
	3	(0:3; 0:4; 0:5)	(0:3; 0:4; 0:5)	(0:4; 0:5; 0:6)	(0:5; 0:6; 0:7)	(0.7,0.8,0.9)
	4	(0:1; 0:2; 0:3)	(0:0; 0:1; 0:2)	(0:5; 0:6; 0:7)	(0:3; 0:4; 0:5)	(0:1; 0:2; 0:3)
	5	(0:7; 0:8; 0:9)	(0:4; 0:5; 0:6)	(0:1; 0:2; 0:3)	(0:3; 0:4; 0:5)	(0:3; 0:4; 0:5)

步骤4

$$
\begin{aligned}
A_{IP} = f &< C_1; (0:40; 0:50; 0:60) >; \; < C_2; (0:70; 0:80; 0:90) >; \\
&< C_3; (0:80; 0:90; 1:00) >; \; < C_4; (0:70; 0:80; 0:90) >; \\
&< C_5; (0:50; 0:60; 0:70) > g:
\end{aligned}
$$

(6-15)-(6-17) A_i $(i = 1; 2; :::; 5)$ J-
E- C- 5-3

步骤5 6-3 6.2.2
A_2 A_2

6.2.4 小结

表 6-3 三种加权相似度的决策结果

	S^J	S^E	S^C
A_1	0.7869	0.8707	0.9974
A_2	0.7976	0.8775	0.9988
A_3	0.6367	0.7546	0.9918
A_4	0.7528	0.8525	0.9964
	$A_2\ \hat{A}\ A_1\ \hat{A}\ A_4\ \hat{A}\ A_3$	$A_2\ \hat{A}\ A_1\ \hat{A}\ A_4\ \hat{A}\ A_3$	$A_2\ \hat{A}\ A_1\ \hat{A}\ A_4\ \hat{A}\ A_3$
	A_2	A_2	A_2

J- E- C-

6.3 基于参数型直觉三角模糊数偏好的多属性大群体决策方法

[53] Li

,Chen Li [144]

6.3.1 三角直觉模糊数和参数型直觉三角模糊数

定义6.6[145] R $\circledR = < (a;b;c);\ {}^{1}{}_{\circledR};\ {}^{o}{}_{\circledR} >$

$$\mu_{\alpha}(x)=\begin{cases}0; & x<a;\\ \frac{(x-a)\mu_{\alpha}}{b-a}; & a\le x\le b;\\ \mu_{\alpha}; & if\ x=b;\\ \frac{(c-x)\mu_{\alpha}}{c-b}; & b\le x\le c;\\ 0; & c<x;\end{cases}$$

$$\nu_{\alpha}(x)=\begin{cases}1; & x<a;\\ \frac{b-x+\nu_{\alpha}(x-a)}{b-a}; & a\le x\le b;\\ \nu_{\alpha}; & if\ x=b;\\ \frac{x-b+\nu_{\alpha}(c-x)}{c-b}; & b\le x\le c;\\ 1; & c<x:\end{cases}$$

μ_{α} ν_{α}

$0\le\mu_{\alpha}\le 1$ $0\le\nu_{\alpha}\le 1$ $0\le\mu_{\alpha}+\nu_{\alpha}\le 1$

matlab 6-2

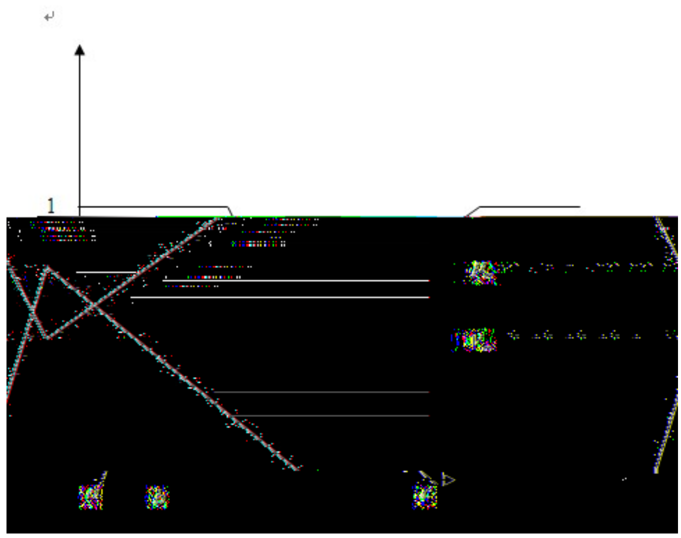

图 6-2 三角直觉模糊数的隶属函数和非隶属函数

6.7

定义6.7 R α 6

$b_1\le a_1\le a_2\le b_2\le a_3\le b_3$ $\alpha=<(a_1;a_2;a_3);(b_1;b_2;b_3)>$

$$^{1}{}_{®}(x)=\begin{cases}0; & x<a_1;\\ \frac{x-a_1}{a_2-a_1}; & a_1\ \cdot\ x\ \cdot\ a_2;\\ \frac{x-a_3}{a_2-a_3}; & a_2\ \cdot\ x\ \cdot\ a_3;\\ 0; & a_3<x;\end{cases} \tag{6-18}$$

$$^{o}{}_{®}(x)=\begin{cases}1; & x<b_1;\\ \frac{x-b_2}{b_1-b_2}; & b_1\ \cdot\ x\ \cdot\ b_2;\\ \frac{x-b_2}{b_3-b_2}; & b_2\ \cdot\ x\ \cdot\ b_3;\\ 1; & b_3<x:\end{cases} \tag{6-19}$$

6-3

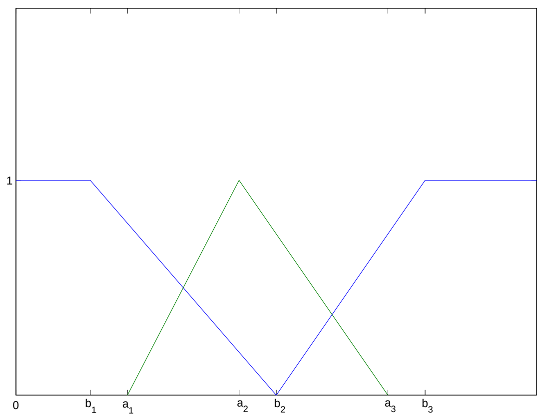

图 6-3 参数型直觉三角模糊数的隶属函数和非隶属函数

$® =< (a_1; a_2; a_3); (b_1; b_2, b_3) >$ $\quad ¯ =< (c_1; c_2; c_3); (d_1; d_2; d_3) >$

°

$$® © ¯ =< (a_1+c_1; a_2+c_2; a_3+c_3); (b_1+d_1; b_2+d_2; b_3+d_3) >;$$

$$® \text{ - } ¯ =< (a_1c_1; a_2c_2; a_3c_3); (b_1d_1; b_2d_2; b_3d_3) >;$$

$$° \text{ - } ® =< (°a_1; °a_2; °a_3); (°b_1; °b_2; °b_3) >;$$

$$\frac{1}{®} =< (\frac{1}{a_1}; \frac{1}{a_2}; \frac{1}{a_3}); (\frac{1}{b_1}; \frac{1}{b_2}; \frac{1}{b_3})$$

定义6.8 α R

α α

$$EV(\alpha) = \frac{E_*(\alpha) + E^*(\alpha)}{2}. \tag{6-20}$$

$$E_*(\alpha) = \frac{b_1 + a_2}{2} + \frac{1}{2}\int_{b_1}^{b_2} h_\alpha(x)dx - \frac{1}{2}\int_{a_1}^{a_2} f_\alpha(x)dx, \tag{6-21}$$

$$E^*(\alpha) = \frac{b_3 + a_2}{2} + \frac{1}{2}\int_{a_2}^{a_3} g_\alpha(x)dx - \frac{1}{2}\int_{b_2}^{b_3} k_\alpha(x)dx. \tag{6-22}$$

定理 6.1. **设**$\alpha =< (a_1, a_2, a_3), (b_1, b_2, b_3) >$ **是实数集**R**的参数型直觉三角模糊数，那么当**

$$f_\alpha(x) = \frac{x - a_1}{a_2 - a_1},$$
$$g_\alpha(x) = \frac{x - a_3}{a_2 - a_3},$$
$$h_\alpha(x) = \frac{x - b_2}{b_1 - b_2},$$
$$k_\alpha(x) = \frac{x - b_2}{b_3 - b_2},$$

且$a_1, a_2, a_3, b_1, b_2, b_3 \in R$**，则**$\alpha$**的期望值为：**

$$EV(\alpha) = \frac{1}{8}(a_1 + 2a_2 + a_3 + b_1 + 2b_2 + b_3). \tag{6-23}$$

证明. 6-21

$$\begin{aligned}
E_*(\alpha) &= \frac{b_1 + a_2}{2} + \frac{1}{2}\int_{b_1}^{b_2} \frac{x - b_2}{b_1 - b_2}dx - \frac{1}{2}\int_{a_1}^{a_2} \frac{x - a_1}{a_2 - a_1}dx \\
&= \frac{b_1 + a_2}{2} + \frac{1}{2(b_1 - b_2)} \times \left(\frac{1}{2}x^2 - b_1 x\right)\Big|_{b_1}^{b_2} - \frac{1}{2(a_2 - a_1)} \times \left(\frac{1}{2}x^2 - a_1 x\right)\Big|_{a_1}^{a_2} \\
&= \frac{b_1 + a_2}{2} + \frac{b_2 - b_1}{4} + \frac{a_1 - a_2}{4} \\
&= \frac{b_2 + a_2 + b_1 + a_1}{4}.
\end{aligned} \tag{6-24}$$

6-22

$$\begin{aligned}
E^*(\alpha) &= \frac{b_3 + a_2}{2} + \frac{1}{2}\int_{a_2}^{a_3} \frac{x - a_3}{a_2 - a_3}dx - \frac{1}{2}\int_{b_2}^{b_3} \frac{x - b_2}{b_3 - b_2}dx \\
&= \frac{b_3 + a_2}{2} + \frac{1}{2(a_2 - a_3)} \times \left(\frac{1}{2}x^2 - a_3 x\right)\Big|_{a_2}^{a_3} - \frac{1}{2(b_3 - b_2)} \times \left(\frac{1}{2}x^2 - b_2 x\right)\Big|_{b_2}^{b_3} \\
&= \frac{b_3 + a_2}{2} + \frac{a_3 - a_2}{4} + \frac{b_2 - b_3}{4} \\
&= \frac{b_2 + a_2 + b_3 + a_3}{4}.
\end{aligned} \tag{6-25}$$

6-24 6-25 6-23

$$EV(®) = \frac{1}{2}(\frac{b_2 + a_2 + b_1 + a_1}{4} + \frac{b_3 + a_2 + b_2 + a_3}{4})$$
$$= \frac{1}{8}(a_1 + 2a_2 + a_3 + b_1 + 2b_2 + b_3):$$

□

$® =< (a_1; a_2; a_3); (b_1; b_2; b_3) >$ $¯ =< (c_1; c_2; c_3); (d_1; d_2; d_3) >$ $® = ¯$, $a_1 = c_1; a_2 = c_2; a_3 = c_3$, $b_1 = d_1; b_2 = d_2; b_3 = d_3$

$$EV(®) = EV(¯):$$

$® > ¯$, i.e. $a_1 > c_1; a_2 > c_2; a_3 > c_3$, $b_1 > d_1; b_2 > d_2; b_3 > d_3$

$$EV(®) > EV(¯):$$

6.3.2 参数型直觉三角模糊数间的相似度

定义6.9 R $S : R^3 £ R^3$ ¡ ! $[0; 1]$ ® ¯ ° $S(®; ¯)$ (P1-P4) $S(®; ¯)$ ® ¯

$(P1)\ 0 \cdot S(®; ¯) \cdot 1;$

$(P2)\ S(®; ¯) = 1;\ if\ ® = ¯;$

$(P3)\ S(®; ¯) = S(¯; ®);$

$(P4)\ S(®; °) \cdot S(®; ¯); S(®; °) \cdot S(¯; °);\ ® \cdot ¯ \cdot °; ®; ¯; °\ 2\ R:$

$®_1 =< (a_{11}; a_{12}; a_{13}); (b_{11}; b_{12}; b_{13}) >$ $®_2 =< (a_{21}; a_{22}, a_{23}); (b_{21}; b_{22}; b_{23}) >$ $®_1$

$®_1$ Hamming Euclidean Hausdor®

$$S^H(®_1; ®_2) = 1 ¡ \frac{\sum_{i=1}^{3} |a_{1i} ¡ a_{2i}|}{3} ¡ \frac{\sum_{i=1}^{3} |b_{1i} ¡ b_{2i}|}{3}; \tag{6-26}$$

$$S^E(®_1; ®_2) = 1 ¡ \sqrt{\frac{\sum_{i=1}^{3} (a_{1i} ¡ a_{2i})^2}{3} + \frac{\sum_{i=1}^{3} (b_{1i} ¡ b_{2i})^2}{3}}; \tag{6-27}$$

$$S^F(\alpha_1,\alpha_2) = 1 - \max\left\{\frac{\sum_{i=1} |a_{1i} - a_{2i}|}{3}, \frac{\sum_{i=1} |b_{1i} - b_{2i}|}{3}\right\}. \qquad (6\text{-}28)$$

定理 6.2. $S^H(\alpha_1,\alpha_2)$, $S^E(\alpha_1,\alpha_2)$**和**$S^F(\alpha_1,\alpha_2)$**是两个参数型直觉三角模糊数**α_1**和**α_2**之间的相似度。**

证明.

P1 $S^H(\alpha_1,\alpha_2) \geq 0$ $S^H(\alpha_1,\alpha_2) \leq 1$

$0 \leq b_1 \leq a_1 \leq a_2 \leq b_2 \leq a_3 \leq b_3 \leq 1$

$$\frac{\sum_{i=1} |a_{1i} - a_{2i}|}{3} + \frac{\sum_{i=1} |b_{1i} - b_{2i}|}{3} \geq 0 \qquad (6\text{-}29)$$

6-29 6-26 $S^H(\alpha_1,\alpha_2) \leq 1$ $0 \leq S^H(\alpha_1,\alpha_2) \leq 1$

P2 $\alpha_1 = \alpha_2$ $a_{11} = a_{21}, a_{12} = a_{22}, a_{13} = a_{23}, b_{11} = b_{21}, b_{12} = b_{22}, b_{13} = b_{23}$ 6-26

$$S^H(\alpha_1,\alpha_2) = 1.$$

(P3)

$$S^H(\alpha_1,\alpha_2) = 1 - \frac{\sum_{i=1} |a_{1i} - a_{2i}|}{3} - \frac{\sum_{i=1} |b_{1i} - b_{2i}|}{3} = S^H(\alpha_2,\alpha_1).$$

P4 $\alpha_3 = <(a_{31}, a_{32}, a_{33}), (b_{31}, b_{32}, b_{33})>$

$\alpha_1 \leq \alpha_2 \leq \alpha_3$ $a_{11} \leq a_{21} \leq a_{31}, a_{12} \leq a_{22} \leq a_{32}, a_{13} \leq a_{23} \leq a_{33}, b_{11} \leq b_{21} \leq b_{31}, b_{12} \leq b_{22} \leq b_{32}, b_{13} \leq b_{23} \leq b_{33}$ 6-26

$$S^H(\alpha_1,\alpha_3) \leq S^H(\alpha_1,\alpha_2), \quad S^H(\alpha_1,\alpha_3) \leq S^H(\alpha_2,\alpha_3)$$

Euclidean Hausdorff

□

6.3.3 参数型直觉三角模糊数偏好的多属性大群体决策方法

$A = \{A_1, A_2, ..., A_p\}$ $C = \{C_1, C_2, ..., C_n\}$

m m

$G = \{G_1, G_2, \ldots, G_m\}$ n

k G_k i A_i

$$v_i^k = \{< C_1, (a_{i11}^k, a_{i12}^k, a_{i13}^k), (b_{i11}^k, b_{i12}^k, b_{i13}^k) >, < C_2, (a_{i21}^k, a_{i22}^k, a_{i23}^k), (b_{i21}^k, b_{i22}^k, b_{i23}^k) >, \ldots, < C_n, (a_{in1}^k, a_{in2}^k, a_{in3}^k), (b_{in1}^k, b_{in2}^k, b_{in3}^k) >\},$$

$i = 1, 2, \ldots, p$; $k = 1, 2, \ldots, m$ $< C_j, (a_{ij1}^k, a_{ij2}^k, a_{ij3}^k)$, $(b_{ij1}^k, b_{ij2}^k, b_{ij3}^k) >$ A_i C_j

A_i

$$\begin{aligned} U_i = \{ & < C_1, (\frac{\sum_{k=1} a_{i11}^k}{m}, \frac{\sum_{k=1} a_{i12}^k}{m}, \frac{\sum_{k=1} a_{i13}^k}{m}), (\frac{\sum_{k=1} b_{i11}^k}{m}, \frac{\sum_{k=1} b_{i12}^k}{m}, \frac{\sum_{k=1} b_{i13}^k}{m}) >, \\ & < C_2, (\frac{\sum_{k=1} a_{i21}^k}{m}, \frac{\sum_{k=1} a_{i22}^k}{m}, \frac{\sum_{k=1} a_{i23}^k}{m}), (\frac{\sum_{k=1} b_{i21}^k}{m}, \frac{\sum_{k=1} b_{i22}^k}{m}, \frac{\sum_{k=1} b_{i23}^k}{m}) >, \ldots, \\ & < C_n, (\frac{\sum_{k=1} a_{in1}^k}{l}, \frac{\sum_{k=1} a_{in2}^k}{m}, \frac{\sum_{k=1} a_{in3}^k}{m}), (\frac{\sum_{k=1} b_{in1}^k}{m}, \frac{\sum_{k=1} b_{in2}^k}{m}, \frac{\sum_{k=1} b_{in3}^k}{m}) >\}, \end{aligned} \tag{6-30}$$

C_j $(j = 1, 2, \ldots, n)$

$w_j = < (a_1, a_2, a_3), (b_1, b_2, b_3) >$ $(j = 1, 2, \ldots, n)$ 6-23

w_j $(j = 1, 2, \ldots, n)$ w_j $(j = 1, 2, \ldots, n)$

,

$$\tilde{w}_j = \frac{EV(w_j)}{\sum_{i=1} EV(w_j)}. \tag{6-31}$$

$< (0.9, 0.9, 1.0), (0.9, 1.0, 1.0) >$

$< (0.0, 0.0, 0.1), (0.0, 0.1, 0.2) >$

A_{IP} 6-26 - 6-

28 i A ¤ ẤAAAA A

表 6-4 语言值与参数型直觉三角模糊数的对照表

(Absolutely-poor)	$<(0{:}0;0{:}0;0{:}0);(0{:}0;0{:}0;0{:}1)>$
(Very-poor)	$<(0{:}0;0{:}0;0{:}1);(0{:}0;0{:}1;0{:}2)>$
(Poor)	$<(0{:}1;0{:}2;0{:}3);(0{:}0;0{:}2;0{:}4)>$
(Medium-poor)	$<(0{:}2;0{:}3;0{:}4);(0{:}1;0{:}3;0{:}5)>$
(Medium)	$<(0{:}4;0{:}5;0{:}6);(0{:}3;0{:}5;0{:}7)>$
(Medium-good)	$<(0{:}5;0{:}6;0{:}7);(0{:}4;0{:}7;0{:}8)>$
(Good)	$<(0{:}7;0{:}8;0{:}9);(0{:}7;0{:}9;1{:}0)>$
(Very-good)	$<(0{:}8;0{:}9;1{:}0);(0{:}8;1{:}0;1{:}0)>$
(Absolutely-good)	$<(0{:}9;0{:}9;1{:}0);(0{:}9;1{:}0;1{:}0)>$

步骤2 (6-30) $A_i\ (i = 1;2;3;4)$

$$\begin{aligned}U_1 = f &< C_1;(0{:}24;0{:}32;0{:}42);(0{:}16;0{:}36;0{:}52)>;<C_2;(0{:}36;0{:}46;0{:}56);\\&(0{:}28;0{:}48;0{:}66)>;<C_3;(0{:}24;0{:}32;0{:}42);(0{:}16;0{:}36;0{:}52)>;\\&<C_4;(0{:}40;0{:}48;0{:}58);(0{:}34;0{:}56;0{:}66)>;<C_5;(0{:}52;0{:}58;0{:}68);\\&(0{:}48;0{:}68;0{:}76)>g;\end{aligned}$$

$$\begin{aligned}U_2 = f &< C_1;(0{:}44;0{:}54;0{:}64);(0{:}36;0{:}60;0{:}74)>;<C_2;(0{:}52;0{:}62;0{:}72);\\&(0{:}46;0{:}68;0{:}80)>;<C_3;(0{:}36;0{:}46;0{:}56);(0{:}26;0{:}50;0{:}66)>;\\&<C_4;(0{:}44;0{:}50;0{:}60);(0{:}40;0{:}56;0{:}66)>;<C_5;(0{:}18;0{:}22;0{:}30);\\&(0{:}16;0{:}28;0{:}38)>g;\end{aligned}$$

$$\begin{aligned}U_3 = f &< C_1;(0{:}34;0{:}42;0{:}52);(0{:}26;0{:}44;0{:}60)>;<C_2;(0{:}28;0{:}36;0{:}44);\\&(0{:}20;0{:}38;0{:}54)>;<C_3;(0{:}42;0{:}52;0{:}28);(0{:}12;0{:}24;0{:}38)>;\\&<C_4;(0{:}46;0{:}52;0{:}62);(0{:}42;0{:}60;0{:}70)>;<C_5;(0{:}62;0{:}34;0{:}58);\\&(0{:}72;0{:}16;0{:}20)>g;\end{aligned}$$

$$\begin{aligned}U_4 = f &< C_1;(0{:}42;0{:}52;0{:}62);(0{:}34;0{:}56;0{:}72)>;<C_2;(0{:}34;0{:}42;0{:}52);\\&(0{:}26;0{:}46;0{:}62)>;<C_3;(0{:}38;0{:}48;0{:}58);(0{:}28;0{:}52;0{:}68)>;\\&<C_4;(0{:}22;0{:}30;0{:}40);(0{:}14;0{:}34;0{:}50)>;<C_5;(0{:}44;0{:}54;0{:}64);\\&(0{:}38;0{:}58;0{:}72>g{:}\end{aligned}$$

表 6-5 5个专家对4个方案的参数型直觉三角模糊数型偏好矢量

A	M	C_1	C_2	C_3
		C_4	C_5	
A_1	1	$<(0.0,0.0,0.1),(0.0,0.1,0.2)>$	$<(0.4,0.5,0.6),(0.3,0.5,0.7)>$	$<(0.5,0.6,0.7),(0.4,0.7,0.8)>$
		$<(0.8,0.9,1.0),(0.8,1.0,1.0)>$	$<(0.5,0.6,0.7),(0.4,0.7,0.8)>$	
	2	$<(0.1,0.2,0.3),(0.0,0.2,0.4)>$	$<(0.2,0.3,0.4),(0.1,0.3,0.5)>$	$<(0.4,0.5,0.6),(0.3,0.5,0.7)>$
		$<(0.5,0.6,0.7),(0.4,0.7,0.8)>$	$<(0.9,0.9,1.0),(0.9,1.0,1.0)>$	
	3	$<(0.4,0.5,0.6),(0.3,0.5,0.7)>$	$<(0.4,0.5,0.6),(0.3,0.5,0.7)>$	$<(0.1,0.2,0.3),(0.0,0.2,0.4)>$
		$<(0.2,0.3,0.4),(0.1,0.3,0.5)>$	$<(0.0,0.0,0.1),(0.0,0.1,0.2)>$	
	4	$<(0.2,0.3,0.4),(0.1,0.3,0.5)>$	$<(0.1,0.2,0.3),(0.0,0.2,0.4)>$	$<(0.0,0.0,0.1),(0.0,0.1,0.2)>$
		$<(0.5,0.6,0.7),(0.4,0.7,0.8)>$	$<(0.7,0.8,0.9),(0.7,0.9,1.0)>$	
	5	$<(0.5,0.6,0.7),(0.4,0.7,0.8)>$	$<(0.7,0.8,0.9),(0.7,0.9,1.0)>$	$<(0.2,0.3,0.4),(0.1,0.3,0.5)>$
		$<(0.0,0.0,0.1),(0.0,0.1,0.2)>$	$<(0.5,0.6,0.7),(0.4,0.7,0.8)>$	
A_2	1	$<(0.4,0.5,0.6),(0.3,0.5,0.7)>$	$<(0.8,0.9,1.0),(0.8,1.0,1.0)>$	$<(0.5,0.6,0.7),(0.4,0.7,0.8)>$
		$<(0.4,0.5,0.6),(0.3,0.5,0.7)>$	$<(0.0,0.0,0.1),(0.0,0.1,0.2)>$	
	2	$<(0.1,0.2,0.3),(0.0,0.2,0.4)>$	$<(0.4,0.5,0.6),(0.3,0.5,0.7)>$	$<(0.8,0.9,1.0),(0.8,1.0,1.0)>$
		$<(0.5,0.6,0.7),(0.4,0.7,0.8)>$	$<(0.1,0.2,0.3),(0.0,0.2,0.4)>$	
	3	$<(0.7,0.8,0.9),(0.7,0.9,1.0)>$	$<(0.5,0.6,0.7),(0.4,0.7,0.8)>$	$<(0.2,0.3,0.4),(0.1,0.3,0.5)>$
		$<(0.1,0.2,0.3),(0.0,0.2,0.4)>$	$<(0.0,0.0,0.1),(0.0,0.1,0.2)>$	
	4	$<(0.5,0.6,0.7),(0.4,0.7,0.8)>$	$<(0.7,0.8,0.9),(0.7,0.9,1.0)>$	$<(0.2,0.3,0.4),(0.1,0.3,0.5)>$
		$<(0.9,0.9,1.0),(0.9,1.0,1.0)>$	$<(0.0,0.0,0.0),(0.0,0.0,0.1)>$	
	5	$<(0.5,0.6,0.7),(0.4,0.7,0.8)>$	$<(0.2,0.3,0.4),(0.1,0.3,0.5)>$	$<(0.4,0.5,0.6),(0.3,0.5,0.7)>$
		$<(0.0,0.0,0.1),(0.0,0.1,0.2)>$	$<(0.8,0.9,1.0),(0.8,1.0,1.0)>$	
A_3	1	$<(0.2,0.3,0.4),(0.1,0.3,0.5)>$	$<(0.1,0.2,0.3),(0.0,0.2,0.4)>$	$<(0.7,0.8,0.9),(0.7,0.9,1.0)>$
		$<(0.4,0.5,0.6),(0.3,0.5,0.7)>$	$<(0.5,0.6,0.7),(0.4,0.7,0.8)>$	
	2	$<(0.1,0.2,0.3),(0.0,0.2,0.4)>$	$<(0.5,0.6,0.7),(0.4,0.7,0.8)>$	$<(0.2,0.3,0.4),(0.1,0.3,0.5)>$
		$<(0.0,0.0,0.0),(0.0,0.0,0.1)>$	$<(0.9,0.9,1.0),(0.9,1.0,1.0)>$	
	3	$<(0.9,0.9,1.0),(0.9,1.0,1.0)>$	$<(0.4,0.5,0.6),(0.3,0.5,0.7)>$	$<(0.5,0.6,0.7),(0.4,0.7,0.8)>$
		$<(0.0,0.0,0.1),(0.0,0.1,0.2)>$	$<(0.2,0.3,0.4),(0.1,0.3,0.5)>$	
	4	$<(0.1,0.2,0.3),(0.0,0.2,0.4)>$	$<(0.0,0.0,0.0),(0.0,0.0,0.1)>$	$<(0.5,0.6,0.7),(0.4,0.7,0.8)>$
		$<(0.4,0.5,0.6),(0.3,0.5,0.7)>$	$<(0.0,0.0,0.1),(0.0,0.1,0.2)>$	
	5	$<(0.4,0.5,0.6),(0.3,0.5,0.7)>$	$<(0.4,0.5,0.6),(0.3,0.5,0.7)>$	$<(0.2,0.3,0.4),(0.1,0.3,0.5)>$
		$<(0.0,0.0,0.1),(0.0,0.1,0.2)>$	$<(0.7,0.8,0.9),(0.7,0.9,1.0)>$	
A_4	1	$<(0.5,0.6,0.7),(0.4,0.7,0.8)>$	$<(0.5,0.6,0.7),(0.4,0.7,0.8)>$	$<(0.4,0.5,0.6),(0.3,0.5,0.7)>$
		$<(0.2,0.3,0.4),(0.1,0.3,0.5)>$	$<(0.8,0.9,1.0),(0.8,1.0,1.0)>$	
	2	$<(0.4,0.5,0.6),(0.3,0.5,0.7)>$	$<(0.2,0.3,0.4),(0.1,0.3,0.5)>$	$<(0.5,0.6,0.7),(0.4,0.7,0.8)>$
		$<(0.0,0.0,0.1),(0.0,0.1,0.2)>$	$<(0.4,0.5,0.6),(0.3,0.5,0.7)>$	
	3	$<(0.2,0.3,0.4),(0.1,0.3,0.5)>$	$<(0.2,0.3,0.4),(0.1,0.3,0.5)>$	$<(0.4,0.5,0.6),(0.3,0.5,0.7)>$
		$<(0.5,0.6,0.7),(0.4,0.7,0.8)>$	$<(0.7,0.8,0.9),(0.7,0.9,1.0)>$	
	4	$<(0.1,0.2,0.3),(0.0,0.2,0.4)>$	$<(0.0,0.0,0.1),(0.0,0.1,0.2)>$	$<(0.5,0.6,0.7),(0.4,0.7,0.8)>$
		$<(0.2,0.3,0.4),(0.1,0.3,0.5)>$	$<(0.1,0.2,0.3),(0.0,0.2,0.4)>$	
	5	$<(0.7,0.8,0.9),(0.7,0.9,1.0)>$	$<(0.4,0.5,0.6),(0.3,0.5,0.7)>$	$<(0.1,0.2,0.3),(0.0,0.2,0.4)>$
		$<(0.2,0.3,0.4),(0.1,0.3,0.5)>$	$<(0.2,0.3,0.4),(0.1,0.3,0.5)>$	

步骤3 $C_j\ (j=1,2,\dots,5)$

$w_1=<C_1,(0.5,0.6,0.7),(0.4,0.7,0.8)>, w_2=<C_2,(0.1,0.2,0.3),(0.0,0.2,0.4)>,$
$w_3=<C_3,(0.5,0.6,0.7),(0.4,0.7,0.8)>, w_4=<C_4,(0.9,0.9,1.0),(0.9,1.0,1.0)>,$
$w_5=<C_5,(0.2,0.3,0.4),(0.1,0.3,0.5)>.$

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5 $\dot{W}=(0.2315,0.0741,0.2315,0.3519,0.1111)$

步骤4

$$A_{IP} = \{\langle C_1, (0.4, 0.5, 0.6), (0.3, 0.5, 0.7)\rangle, \langle C_2, (0.7, 0.8, 0.9), (0.7, 0.9, 1.0)\rangle,$$
$$\langle C_3, (0.8, 0.9, 1.0), (0.8, 1.0, 1.0)\rangle, \langle C_4, (0.7, 0.8, 0.9), (0.7, 0.9, 1.0)\rangle,$$
$$\langle C_5, (0.5, 0.6, 0.7), (0.4, 0.7, 0.8)\rangle\}.$$

6-32 - 6-34 $A_i\ (i = 1, 2, \ldots, 5)$

6-6

表 6-6 三种加权相似度的决策结果

	S^H	S^E	S^F
A_1	0.3433	0.5325	0.6574
A_2	0.4667	0.6181	0.7172
A_3	0.2845	0.4887	0.6306
A_4	0.3535	0.5388	0.6611
	$A_2 \succ A_4 \succ A_1 \succ A_3$	$A_2 \succ A_4 \succ A_1 \succ A_3$	$A_2 \succ A_4 \succ A_1 \succ A_3$
	A_2	A_2	A_2

步骤5 6-6

A_2

6.3.5 小结

Hamming Euclidean

Hausdorff

6.4 本章小结

7 基于梯形模糊数偏好的多属性大群体决策方法

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[41] [38]

7.1 改进的TOPSIS方法在梯形模糊偏好多属性决策问题中的应用

[147]-[148]

[149] [150]-[151]

[152]

OWA

[153]

[154]

TOPSIS

7.1.1 梯形模糊数及其性质

定义7.1 ® = (*a*; *b*; *c*; *d*) *R* 0 < *a* · *b* · *c* · *d* < *1* ® *¹*®(*x*)

$$
{}^{1}{}_{®}(x) = \begin{cases} 0; & x < a; \\ \frac{x-a}{b-a}; & a \cdot x \cdot b; \\ 1; & b \cdot x \cdot c; \\ \frac{x-d}{c-d}; & c \cdot x \cdot d; \\ 0; & d < x: \end{cases} \tag{7-1}
$$

7-1

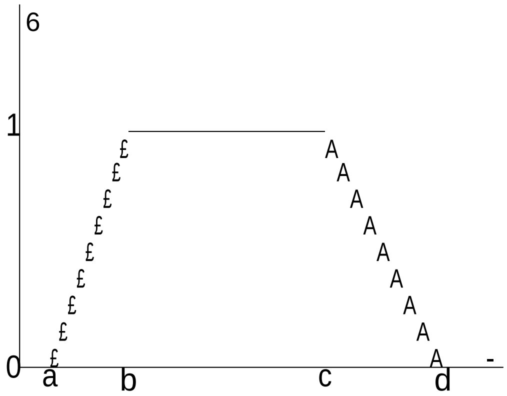

图 7-1 梯形模糊数的隶属函数图

$b = c$ ¯

(¯; ¯; ¯; ¯) [¯; °]

(¯; ¯; °; °)

® = $(a_1; a_2; a_3; a_4)$ ¯ = $(b_1; b_2; b_3; b_4)$ °

$$
® © ¯ = (a_1 + b_1; a_2 + b_2; a_3 + b_3; a_4 + b_4);
$$

$$
® - ¯ = (a_1 b_1; a_2 b_2; a_3 b_3; a_4 b_4);
$$

$$
° - ® = (° a_1; ° a_2; ° a_3; ° a_4);
$$

$$
\frac{1}{®} = (\frac{1}{a_1}; \frac{1}{a_2}; \frac{1}{a_3}; \frac{1}{a_4}):
$$

定义7.2 ® = $(a; b; c; d)$ R $0 < a \cdot b \cdot c \cdot d <$ 1 ®

$$
EV(®) = \frac{1}{2}[E_{-}(®) + E_{+}(®)]; \tag{7-2}
$$

$E_{-}(®) = \frac{a+b}{2}$ $E_{+}(®) = \frac{c+d}{2}$

定理 7.1. 设® = $(a; b; c; d)$ **是实数集**R**的梯形模糊数，且**$a; b; c; d$ 2 R**，那么当**

$$
E_{-}(®) = \frac{a+b}{2};
$$

$$
E_{+}(®) = \frac{c+d}{2};
$$

则®的期望值为：

$$EV(®) = \frac{1}{4}(a + b + c + d): \qquad (7\text{-}3)$$

7.1

定义7.3 ® = $(a_1; a_2; a_3; a_4)$ ¯ = $(b_1; b_2; b_3; b_4)$

$$S_O = \sqrt{\sum_i (a_i \;\; i \;\; b_i)^2}: \qquad (7\text{-}4)$$

7.3

$(P1)$ $0 \;\cdot\; S_O(®; ¯) \;\cdot\; 1;$

$(P2)$ $S_O(®; ¯) = 1; \;\; if \; ® = ¯;$

$(P3)$ $S_O(®; ¯) = S(¯; ®);$

$(P4)$ $S_O(®; °) \;\cdot\; S_O(®; ¯); \; S(®; °) \;\cdot\; S(¯; °); \;\; ® \cdot ¯ \cdot °; ®; ¯; ° \; 2 \; R:$

7.1.2 改进TOPSIS方法及应用

[158],[159] f g

7-1

$$t_{ij} = \frac{1}{m}(t_{ij}^1 + t_{ij}^2 + \cdots + t_{ij}^m); \tag{7-5}$$

$$w_j = \frac{1}{m}(w_j^1 + w_j^2 + \cdots + w_j^m). \tag{7-6}$$

T W $t_{ij} = (a_{ij}, b_{ij}, c_{ij}, d_{ij})$ $W = (w_1, w_2, \cdots, w_n)$

$$T = (t_{ij})_{n\times p} = \begin{pmatrix} t_{11} & t_{12} & \cdots & t_{1p} \\ t_{21} & t_{22} & \cdots & t_{2p} \\ \vdots & \vdots & \vdots & \vdots \\ t_{n1} & t_{n2} & \cdots & t_{np} \end{pmatrix}.$$

TOPSIS

步骤1 $A_i\ (i = 1, 2, \cdots, p)$ $C_j\ (j = 1, 2, \cdots, n)$ 7-1

表 7-1 语言值与梯形模糊数的对照表

t_0	(0.0, 0.0, 0.1, 0.1)
t_1	(0.0, 0.1, 0.2, 0.3)
t_2	(0.2, 0.2, 0.3, 0.4)
t_3	(0.3, 0.3, 0.4, 0.5)
t_4	(0.6, 0.7, 0.8, 0.9)
t_5	(0.7, 0.8, 0.9, 1.0)
t_6	(0.9, 0.9, 1.0, 1.0)

步骤2 7-5 7-6 $A_i\ (i = 1, 2, \cdots, p)$ $C_j\ (j = 1, 2, \cdots, n)$ T^U

步骤3 7-3 6-31 n $\hat{W} = (\hat{w}_1, \hat{w}_2, \cdots, \hat{w}_n)$

步骤4 OWA $A_i\ (i = 1, 2, \cdots, p)$

$$\hat{T}^U = \begin{pmatrix} \hat{w}_1 t_{11} & \hat{w}_1 t_{12} & \cdots & \hat{w}_1 t_{1p} \\ \hat{w}_2 t_{21} & \hat{w}_2 t_{22} & \cdots & \hat{w}_2 t_{2p} \\ \vdots & \vdots & \vdots & \vdots \\ \hat{w}_n t_{n1} & \hat{w}_n t_{n2} & \cdots & \hat{w}_n t_{np} \end{pmatrix}.$$

步骤5 V^+ V^-

$$V^+ = Max\{w_1 T(1), w_2 T(2), \ldots, w_n T(n)\}; \tag{7-7}$$

$$V^- = Min\{w_1 T(1), w_2 T(2), \ldots, w_n T(n)\}; \tag{7-8}$$

$T(i)$ T i 7-4

S_O^+ S_O^-

$$S_O^+ = \sum_{j=1} \sqrt{\sum_{i=1} (t_{ij} - v_{ij}^+)^2}; \tag{7-9}$$

$$S_O^- = \sum_{j=1} \sqrt{\sum_{i=1} (t_{ij} - v_{ij}^-)^2}. \tag{7-10}$$

步骤6 A_i V^+ :

$$RS_O^+ = \frac{S_O^+}{S_O^+ + S_O^-}. \tag{7-11}$$

步骤7

7.1.3 案例分析

A_1 A_2 A_3 A_4

6 G_1 G_2 G_3 G_4 G_5 G_6

(1) (C_1) (2) (C_2) (3)

(C_3) (4) (C_4) 6 $\{$

$\}$ $\{t_0, t_1, t_2, t_3, t_4, t_5, t_6\}$

$$T^1 = \begin{pmatrix} t_3 & t_1 & t_5 & t_4 \\ t_4 & t_4 & t_5 & t_4 \\ t_5 & t_5 & t_4 & t_1 \\ t_3 & t_5 & t_1 & t_5 \end{pmatrix}; \quad T^2 = \begin{pmatrix} t_4 & t_4 & t_4 & t_1 \\ t_4 & t_3 & t_5 & t_4 \\ t_4 & t_5 & t_5 & t_1 \\ t_1 & t_4 & t_3 & t_4 \end{pmatrix}; \quad T^3 = \begin{pmatrix} t_3 & t_4 & t_5 & t_4 \\ t_4 & t_5 & t_5 & t_3 \\ t_1 & t_4 & t_3 & t_3 \\ t_4 & t_1 & t_1 & t_5 \end{pmatrix};$$

$$T^4 = \begin{pmatrix} t_2 & t_1 & t_3 & t_4 \\ t_4 & t_6 & t_1 & t_4 \\ t_3 & t_5 & t_5 & t_1 \\ t_2 & t_4 & t_1 & t_5 \end{pmatrix}; \quad T^5 = \begin{pmatrix} t_1 & t_4 & t_5 & t_1 \\ t_4 & t_2 & t_2 & t_4 \\ t_4 & t_2 & t_6 & t_1 \\ t_1 & t_3 & t_3 & t_0 \end{pmatrix}; \quad T^6 = \begin{pmatrix} t_2 & t_4 & t_5 & t_4 \\ t_4 & t_4 & t_5 & t_3 \\ t_1 & t_3 & t_3 & t_5 \\ t_4 & t_0 & t_1 & t_4 \end{pmatrix};$$

$$T^C = \begin{pmatrix} t_5 & t_2 & t_3 & t_2 \\ t_6 & t_6 & t_1 & t_5 \\ t_4 & t_5 & t_2 & t_1 \\ t_4 & t_4 & t_4 & t_4 \\ t_6 & t_2 & t_5 & t_3 \\ t_2 & t_3 & t_0 & t_5 \end{pmatrix}.$$

7.1.2 TOPSIS

步骤1

$$T^1 = \begin{pmatrix} (0.3, 0.3, 0.4, 0.5) & (0.0, 0.1, 0.2, 0.3) & (0.7, 0.8, 0.9, 1.0) & (0.6, 0.7, 0.8, 0.9) \\ (0.6, 0.7, 0.8, 0.9) & (0.6, 0.7, 0.8, 0.9) & (0.7, 0.8, 0.9, 1.0) & (0.6, 0.7, 0.8, 0.9) \\ (0.7, 0.8, 0.9, 1.0) & (0.7, 0.8, 0.9, 1.0) & (0.6, 0.7, 0.8, 0.9) & (0.0, 0.1, 0.2, 0.3) \\ (0.3, 0.3, 0.4, 0.5) & (0.7, 0.8, 0.9, 1.0) & (0.0, 0.1, 0.2, 0.3) & (0.7, 0.8, 0.9, 1.0) \end{pmatrix};$$

$$T^2 = \begin{pmatrix} (0.6, 0.7, 0.8, 0.9) & (0.6, 0.7, 0.8, 0.9) & (0.6, 0.7, 0.8, 0.9) & (0.0, 0.1, 0.2, 0.3) \\ (0.6, 0.7, 0.8, 0.9) & (0.3, 0.3, 0.4, 0.5) & (0.7, 0.8, 0.9, 1.0) & (0.6, 0.7, 0.8, 0.9) \\ (0.6, 0.7, 0.8, 0.9) & (0.7, 0.8, 0.9, 1.0) & (0.7, 0.8, 0.9, 1.0) & (0.0, 0.1, 0.2, 0.3) \\ (0.0, 0.1, 0.2, 0.3) & (0.6, 0.7, 0.8, 0.9) & (0.3, 0.3, 0.4, 0.5) & (0.6, 0.7, 0.8, 0.9) \end{pmatrix};$$

$$T^3 = \begin{pmatrix} (0.3, 0.3, 0.4, 0.5) & (0.6, 0.7, 0.8, 0.9) & (0.7, 0.8, 0.9, 1.0) & (0.6, 0.7, 0.8, 0.9) \\ (0.6, 0.7, 0.8, 0.9) & (0.7, 0.8, 0.9, 1.0) & (0.7, 0.8, 0.9, 1.0) & (0.3, 0.3, 0.4, 0.5) \\ (0.0, 0.1, 0.2, 0.3) & (0.6, 0.7, 0.8, 0.9) & (0.3, 0.3, 0.4, 0.5) & (0.3, 0.3, 0.4, 0.5) \\ (0.6, 0.7, 0.8, 0.9) & (0.0, 0.1, 0.2, 0.3) & (0.0, 0.1, 0.2, 0.3) & (0.7, 0.8, 0.9, 1.0) \end{pmatrix};$$

$$T^4 = \begin{pmatrix} (0.2, 0.2, 0.3, 0.4) & (0.0, 0.1, 0.2, 0.3) & (0.3, 0.3, 0.4, 0.5) & (0.6, 0.7, 0.8, 0.9) \\ (0.6, 0.7, 0.8, 0.9) & (0.9, 0.9, 1.0, 1.0) & (0.0, 0.1, 0.2, 0.3) & (0.6, 0.7, 0.8, 0.9) \\ (0.3, 0.3, 0.4, 0.5) & (0.7, 0.8, 0.9, 1.0) & (0.7, 0.8, 0.9, 1.0) & (0.0, 0.1, 0.2, 0.3) \\ (0.2, 0.2, 0.3, 0.4) & (0.6, 0.7, 0.8, 0.9) & (0.0, 0.1, 0.2, 0.3) & (0.7, 0.8, 0.9, 1.0) \end{pmatrix};$$

$$T^5 = \begin{pmatrix} (0.0, 0.1, 0.2, 0.3) & (0.6, 0.7, 0.8, 0.9) & (0.7, 0.8, 0.9, 1.0) & (0.0, 0.1, 0.2, 0.3) \\ (0.6, 0.7, 0.8, 0.9) & (0.2, 0.2, 0.3, 0.4) & (0.2, 0.2, 0.3, 0.4) & (0.6, 0.7, 0.8, 0.9) \\ (0.6, 0.7, 0.8, 0.9) & (0.2, 0.2, 0.3, 0.4) & (0.9, 0.9, 1.0, 1.0) & (0.0, 0.1, 0.2, 0.3) \\ (0.0, 0.1, 0.2, 0.3) & (0.3, 0.3, 0.4, 0.5) & (0.3, 0.3, 0.4, 0.5) & (0.0, 0.0, 0.1, 0.1) \end{pmatrix};$$

$$T^{6} = \begin{pmatrix} (0.2, 0.2, 0.3, 0.4) & (0.6, 0.7, 0.8, 0.9) & (0.7, 0.8, 0.9, 1.0) & (0.6, 0.7, 0.8, 0.9) \\ (0.6, 0.7, 0.8, 0.9) & (0.6, 0.7, 0.8, 0.9) & (0.7, 0.8, 0.9, 1.0) & (0.3, 0.3, 0.4, 0.5) \\ (0.0, 0.1, 0.2, 0.3) & (0.3, 0.3, 0.4, 0.5) & (0.3, 0.3, 0.4, 0.5) & (0.7, 0.8, 0.9, 1.0) \\ (0.6, 0.7, 0.8, 0.9) & (0.0, 0.0, 0.1, 0.1) & (0.0, 0.1, 0.2, 0.3) & (0.6, 0.7, 0.8, 0.9) \end{pmatrix};$$

$$T^{C} = \begin{pmatrix} (0.7, 0.8, 0.9, 1.0) & (0.2, 0.2, 0.3, 0.4) & (0.3, 0.3, 0.4, 0.5) & (0.2, 0.2, 0.3, 0.4) \\ (0.9, 0.9, 1.0, 1.0) & (0.9, 0.9, 1.0, 1.0) & (0.0, 0.1, 0.2, 0.3) & (0.7, 0.8, 0.9, 1.0) \\ (0.6, 0.7, 0.8, 0.9) & (0.7, 0.8, 0.9, 1.0) & (0.2, 0.2, 0.3, 0.4) & (0.0, 0.1, 0.2, 0.3) \\ (0.6, 0.7, 0.8, 0.9) & (0.6, 0.7, 0.8, 0.9) & (0.6, 0.7, 0.8, 0.9) & (0.6, 0.7, 0.8, 0.9) \\ (0.9, 0.9, 1.0, 1.0) & (0.2, 0.2, 0.3, 0.4) & (0.7, 0.8, 0.9, 1.0) & (0.3, 0.3, 0.4, 0.5) \\ (0.2, 0.2, 0.3, 0.4) & (0.3, 0.3, 0.4, 0.5) & (0.0, 0.0, 0.1, 0.1) & (0.7, 0.8, 0.9, 1.0) \end{pmatrix}.$$

步骤2 7-5 7-6 $A_i\ (i = 1, 2, \ldots, 4)$ $C_j\ (j = 1, 2, \ldots, 4)$ T^U

$$T^{U} = \begin{pmatrix} (0.3, 0.3, 0.4, 0.5) & (0.4, 0.5, 0.6, 0.7) & (0.6, 0.7, 0.8, 0.9) & (0.4, 0.5, 0.6, 0.7) \\ (0.6, 0.7, 0.8, 0.9) & (0.6, 0.6, 0.7, 0.8) & (0.5, 0.6, 0.7, 0.8) & (0.5, 0.6, 0.7, 0.8) \\ (0.4, 0.5, 0.6, 0.7) & (0.5, 0.6, 0.7, 0.8) & (0.6, 0.6, 0.7, 0.8) & (0.2, 0.3, 0.4, 0.5) \\ (0.3, 0.4, 0.4, 0.6) & (0.4, 0.4, 0.5, 0.7) & (0.1, 0.2, 0.3, 0.4) & (0.6, 0.6, 0.7, 0.9) \end{pmatrix}.$$

$$\begin{aligned} \tilde{W}^G = \{ &< C_1, (0.65, 0.70, 0.80, 0.8667) >, < C_2, (0.4833, 0.5167, 0.6167, 0.70) >, \\ &< C_3, (0.30, 0.35, 0.45, 0.5333) >, < C_4, (0.4167, 0.4833, 0.5833, 0.6833) > \}. \end{aligned}$$

步骤3 7-3

$$EV(W^G) = (0.7542, 0.5792, 0.4083, 0.5416),$$

6-31 4

$$\tilde{W}^G = (0.3303, 0.2537, 0.1788, 0.2372),$$

步骤4 OWA $A_i\ (i = 1, 2, \ldots, p)$

$$T^{1U}=\begin{pmatrix} 0.3303(0.3,0.3,0.4,0.5) & 0.2537(0.4,0.5,0.6,0.7) & 0.1788(0.6,0.7,0.8,0.9) & 0.2372(0.4,0.5,0.6,0.7) \\ 0.3303(0.6,0.7,0.8,0.9) & 0.2537(0.6,0.6,0.7,0.8) & 0.1788(0.5,0.6,0.7,0.8) & 0.2372(0.5,0.6,0.7,0.8) \\ 0.3303(0.4,0.5,0.6,0.7) & 0.2537(0.5,0.6,0.7,0.8) & 0.1788(0.6,0.6,0.7,0.8) & 0.2372(0.2,0.3,0.4,0.5) \\ 0.3303(0.3,0.4,0.4,0.6) & 0.2537(0.4,0.4,0.5,0.7) & 0.1788(0.1,0.2,0.3,0.4) & 0.2372(0.6,0.6,0.7,0.9) \end{pmatrix}$$

$$=\begin{pmatrix} (0.1,0.1,0.1,0.2) & (0.1,0.2,0.2,0.2) & (0.2,0.2,0.3,0.3) & (0.1,0.2,0.2,0.2) \\ (0.2,0.2,0.2,0.2) & (0.1,0.2,0.2,0.2) & (0.1,0.1,0.1,0.2) & (0.1,0.1,0.2,0.2) \\ (0.1,0.1,0.1,0.1) & (0.1,0.1,0.1,0.1) & (0.1,0.1,0.1,0.1) & (0.0,0.0,0.1,0.1) \\ (0.1,0.1,0.1,0.1) & (0.1,0.1,0.1,0.1) & (0.0,0.0,0.1,0.1) & (0.1,0.2,0.2,0.2) \end{pmatrix}.$$

步骤5 7-7 7-8 V^+ V^-

$$\begin{aligned} V^+ &= \{<C_1,(0.2,0.2,0.2,0.2)>,<C_2,(0.1,0.2,0.2,0.2)>, \\ &\quad <C_3,(0.2,0.2,0.3,0.3)>,<C_4,(0.1,0.2,0.2,0.2)>\}. \\ V^- &= \{<C_1,(0.1,0.1,0.1,0.1)>,<C_2,(0.1,0.1,0.1,0.1)>, \\ &\quad <C_3,(0.0,0.0,0.1,0.1)>,<C_4,(0.0,0.0,0.1,0.1)>\}. \end{aligned}$$

7-9 7-10

S_O^+ S_O^-

$$S_{O1}^+=0.1463,\ S_{O2}^+=0.2678,\ S_{O3}^+=0.8476,\ S_{O4}^+=0.7641,$$
$$S_{O1}^-=0.8533,\ S_{O2}^-=0.7318,\ S_{O3}^-=0.1510,\ S_{O3}^-=0.2360.$$

步骤6 7-11 V^+ :

$$RS_{O1}^+=0.1464,\ RS_{O2}^+=0.2679,\ RS_{O3}^+=0.8489,\ RS_{O4}^+=0.7640.$$

步骤7 A_3

7.1.4 小结

TOPSIS

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7.2 基于参数型直觉梯形模糊数偏好的多属性决策方法

[36]

•

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$(a_1, a_2, a_3, a_4), (b_1, b_2, b_3, b_4) >$

$$\mu_{\alpha}(x) = \begin{cases} 0, & x < a_1, \\ \frac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2, \\ 1, & a_2 < x < a_3, \\ \frac{x-a_4}{a_3-a_4}, & a_3 \le x \le a_4, \\ 0, & a_4 < x, \end{cases} \tag{7-12}$$

$$\nu_{\alpha}(x) = \begin{cases} 0, & x < b_1, \\ \frac{x-b_1}{b_2-b_2}, & b_1 \le x \le b_2, \\ 1, & b_2 < x < b_3, \\ \frac{x-b_3}{b_4-b_3}, & b_3 \le x \le b_4, \\ 0, & b_4 < x, \end{cases} \tag{7-13}$$

7.4

7-2

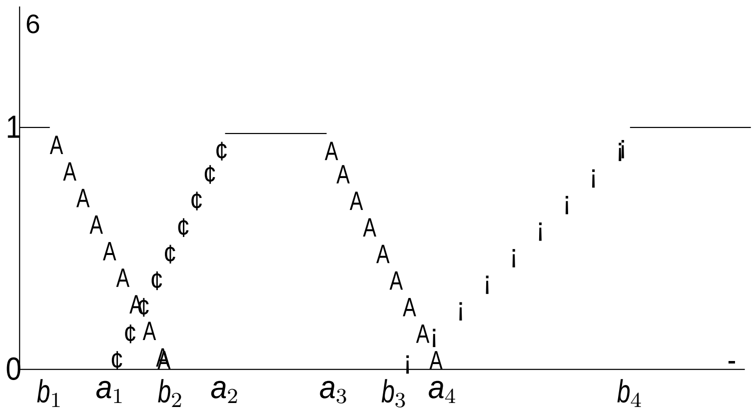


图 7-2 参数型直觉梯形模糊数的隶属函数和非隶属函数图

$\alpha_1 =< (a_{11}, a_{12}, a_{13}, a_{14}), (b_{11}, b_{12}, b_{13}, b_{14}) >$ $\alpha_2 =< (a_{21}, a_{22}, a_{23}, a_{24}), (b_{21}, b_{22}, b_{23}, b_{24}) >$,

。

$$\alpha_1 + \alpha_2 =< (a_{11} + a_{21}, a_{12} + a_{22}, a_{13} + a_{23}, a_{14} + a_{24}), (b_{11} + b_{21}, b_{12} + b_{22}, b_{13} + b_{23}, b_{14} + b_{24}) > .$$

$$\lambda\alpha_1 =< (\lambda a_{11}, \lambda a_{12}, \lambda a_{13}, \lambda a_{14}), (\lambda b_{11}, \lambda b_{12}, \lambda b_{13}, \lambda b_{14}) > .$$

Ye [40]

定义7.5 ® =< $(a_1; a_2; a_3; a_4); (b_1; b_2; b_3; b_4)$ > R

$¹_®(x)$ $º_®(x)$ 6-2 6-3 :

$$EV(®) = \frac{E_*(®) + E^*(®)}{2}; \tag{7-14}$$

$$E_*(®) = \frac{b_1 + a_2}{2} + \frac{1}{2}\int_{b_1}^{b_2} h_®(x)dx \; ¡ \; \frac{1}{2}\int_{a_1}^{a_2} f_®(x)dx; \tag{7-15}$$

$$E^*(®) = \frac{b_4 + a_3}{2} + \frac{1}{2}\int_{a_3}^{a_4} g_®(x)dx \; ¡ \; \frac{1}{2}\int_{b_3}^{b_4} k_®(x)dx; \tag{7-16}$$

定理 7.2. 设®是实数集R的参数型直觉梯形模糊数，特别地当

$$f_®(x) = \frac{x ¡ a_1}{a_2 ¡ a_1};$$
$$g_®(x) = \frac{x ¡ a_4}{a_3 ¡ a_4};$$
$$h_®(x) = \frac{x ¡ b_2}{b_1 ¡ b_2};$$
$$k_®(x) = \frac{x ¡ b_3}{b_4 ¡ b_3};$$

则®的期望值为：

$$EV(®) = \frac{1}{8}(a_1 + a_2 + a_3 + a_4 + b_1 + b_2 + b_3 + b_4): \tag{7-17}$$

证明. 7-15

$$E_*(®) = \frac{b_1 + a_2}{2} + \frac{1}{2}\int_{b_1}^{b_2} \frac{x ¡ b_2}{b_1 ¡ b_2}dx \; ¡ \; 1$$

7-18 7-19 7-16

$$EV(®) = \frac{1}{2}(\frac{b_2 + a_2 + b_1 + a_1}{4} + \frac{b_3 + a_3 + b_4 + a_4}{4})$$
$$= \frac{1}{8}(a_1 + a_2 + a_3 + a_4 + b_1 + b_2 + b_3 + b_4):$$

□

® ¯ $EV(®)$ ¸ $EV(¯)$, ® ¸ ¯

2.2

J- E- C-

7.2.2 参数型直觉梯形模糊数偏好多属性决策方法

p $A = fA_1; A_2; :::; A_pg$,

n $(C = fC_1; C_2; :::; C_ng)$ p i

j

$$v^i_j =< (a^i_{j_1}; a^i_{j_2}; a^i_{j_3}; a^i_{j_4}); (b^i_{j_1}; b^i_{j_2}; b^i_{j_3}; b^i_{j_4}) > : \quad (7\text{-}20)$$

$V^i = (v^i_1; v^i_2; :::; v^i_n)$ A i

7-17 $E^i = (e^i_1; e^i_2; :::; e^i_n)$

A

定义7.6 $E^1 = (e^1_1; e^1_2; :::; e^1_n)$ $E^2 = (e^2_1; e^2_2; :::; e^2_n)$

, E^1 E^2

$S^{12} = S(E^1; E^2) = \frac{1}{2}\frac{2E^1E^2}{kE ; E^2 ; E}$ 2(2)95 2 005] 12 005 11 810 [(2)] 29 3061296 2216 0030

证明. , $S(E^1;E^2)$ 3.1 (P2-P3) $S(E^1;E^2)$ (P1).

(P1)$S(E^1;E^2) \geq 0$ $S(E^1;E^2) \leq 1$

$$(e_i^1)^2 + (e_i^2)^2 \geq 2e_i^1 e_i^2; \quad \sqrt{\sum_{i=1}(e_i^1)^2}\sqrt{\sum_{i=1}(e_i^2)^2} \geq \sum_{i=1} e_i^1 e_i^2.$$

7-21

$$S(E^1;E^2) = \frac{1}{2}\frac{\sum_{i=1} e_i^1 e_i^2}{\sum_{i=1}(e_i^1)^2 + \sum_{i=1}(e_i^2)^2} + \frac{1}{2}\frac{\sum_{i=1} e_i^1 e_i^2}{\sqrt{\sum_{i=1}(e_i^1)^2}\sqrt{\sum_{i=1}(e_i^2)^2}}$$
$$\leq \frac{1}{2} + \frac{1}{2} \leq 1.$$

$0 \leq S(E^1;E^2) \leq 1$

□

A $E^P = (e_1^P; e_2^P; \ldots; e_n^P)$ $E^N = (e_1^N; e_2^N; \ldots; e_n^N)$ A i S^{Pi}, S^{Ni}

$$RS^i = \frac{S^{Pi}}{S^{Ni}} \tag{7-22}$$

$$k = \max_{1\leq i\leq m}\{RS^i\}. \tag{7-23}$$

7.2.3 案例分析

$A = \{(A_1,$ $); (A_2,$ $); (A_3,$ $); (A_4,$ $); (A_5,$ $); (A_6,$ $); (A_7,$ $); (A_8,$ $); (A_9,$ $)\}$ $C = \{(C_1,$ $); (C_2,$ $); (C_3,$ $)$ $(C_4$ $)\}$ 7-2.

:

表 7-2 语言值与参数型直觉梯形模糊数对照表

1	< (0:0; 0:0; 0:0; 0:0); (0:0; 0:0; 0:0; 0:0) >
2	< (0:0; 0:0; 0:1; 0:2); (0:0; 0:0; 0:1; 0:2) >
3	< (0:0; 0:1:0:2; 0:3); (0:0; 0:1; 0:2; 0:3) >
4	< (0:1; 0:2; 0:3; 0:4); (0:0; 0:2; 0:3; 0:5) >
5	< (0:3; 0:4; 0:5; 0:6); (0:2; 0:4; 0:5; 0:7) >
6	< (0:5; 0:6; 0:7; 0:8); (0:4; 0:6; 0:7; 0:9) >
7	< (0:7; 0:8; 0:9; 1:0); (0:7; 0:8; 0:9; 1:0) >
8	< (0:8; 0:9; 1:0; 1:0); (0:8; 0:9; 1:0; 1:0) >
9	< (1:0; 1:0; 1:0; 1:0); (1:0; 1:0; 1:0; 1:0) >

表 7-3 决策方案的参数型直觉梯形模糊偏好矢量

A	c_1	c_2
	c_3	c_4
1	$<(0.0, 0.0, 0.1, 0.2), (0.0, 0.0, 0.1, 0.2)>$	$<(0.3, 0.4, 0.5, 0.6), (0.2, 0.4, 0.5, 0.7)>$
	$<(0.5, 0.6, 0.7, 0.8), (0.4, 0.6, 0.7, 0.9)>$	$<(0.8, 0.9, 1.0, 1.0), (0.8, 0.9, 1.0, 1.0)>$
2	$<(0.0, 0.1.0.2, 0.3), (0.0, 0.1.0.2, 0.3)>$	$<(0.1, 0.2, 0.3, 0.4), (0.0, 0.2, 0.3, 0.5)>$
	$<(0.3, 0.4, 0.5, 0.6), (0.2, 0.4, 0.5, 0.7)>$	$<(1.0, 1.0, 1.0, 1.0), (1.0, 1.0, 1.0, 1.0)>$
3	$<(0.3, 0.4, 0.5, 0.6), 0.2, 0.4, 0.5, 0.7)>$	$<(0.3, 0.4, 0.5, 0.6), (0.2, 0.4, 0.5, 0.7)>$
	$<(0.1, 0.2, 0.3, 0.4), (0.0, 0.2, 0.3, 0.5)>$	$<(0.1, 0.2, 0.3, 0.4), (0.0, 0.2, 0.3, 0.5)>$
4	$<(0.3, 0.4, 0.5, 0.6), (0.2, 0.4, 0.5, 0.7)>$	$<(0.0, 0.1, 0.2, 0.3), (0.0, 0.1, 0.2, 0.3)>$
	$<(0.0, 0.0, 0.1, 0.2), (0.0, 0.0, 0.1, 0.2)>$	$<(0.5, 0.6, 0.7, 0.8), (0.4, 0.6, 0.7, 0.9)>$
5	$<(0.3, 0.4, 0.5, 0.6), (0.2, 0.4, 0.5, 0.7)>$	$<(0.5, 0.6, 0.7, 0.8), (0.4, 0.6, 0.7, 0.9)>$
	$<(0.1, 0.2, 0.3, 0.4), (0.0, 0.2, 0.3, 0.5)>$	$<(0.0, 0.0, 0.1, 0.2), (0.0, 0.0, 0.1, 0.2)>$
6	$<(0.8, 0.9, 1.0, 1.0), (0.8, 0.9, 1.0, 1.0)>$	$<(0.0, 0.0, 0.1, 0.2), (0.0, 0.0, 0.1, 0.2)>$
	$<(0.3, 0.4, 0.5, 0.6), (0.2, 0.4, 0.5, 0.7)>$	$<(0.1, 0.2, 0.3, 0.4), (0.0, 0.2, 0.3, 0.5)>$
7	$<(0.7, 0.8, 0.9, 1.0), (0.7, 0.8, 0.9, 1.0)>$	$<(0.0, 0.1.0.2, 0.3), (0.0, 0.1.0.2, 0.3)>$
	$<(0.1, 0.2, 0.3, 0.4), (0.0, 0.2, 0.3, 0.5)>$	$<(0.8, 0.9, 1.0, 1.0), (0.8, 0.9, 1.0, 1.0)>$
8	$<(0.5, 0.6, 0.7, 0.8), (0.4, 0.6, 0.7, 0.9)>$	$<(0.5, 0.6, 0.7, 0.8), (0.4, 0.6, 0.7, 0.9)>$
	$<(0.3, 0.4, 0.5, 0.6), (0.2, 0.4, 0.5, 0.7)>$	$<(0.0, 0.0, 0.1, 0.2), (0.0, 0.0, 0.1, 0.2)>$
9	$<(0.1, 0.2, 0.3, 0.4), (0.0, 0.2, 0.3, 0.5)>$	$<(0.0, 0.0, 0.1, 0.2), (0.0, 0.0, 0.1, 0.2)>$
	$<(0.1, 0.2, 0.3, 0.4), (0.0, 0.2, 0.3, 0.5)>$	$<(0.0, 0.0, 0.0, 0.0), (0.0, 0.0, 0.0, 0.0)>$
P	$<(0.0, 0.0, 0.1, 0.2), (0.0, 0.0, 0.1, 0.2)>$	$<(0.8, 0.9, 1.0, 1.0), (0.8, 0.9, 1.0, 1.0)>$
	$<(0.7, 0.8, 0.9, 1.0), (0.7, 0.8, 0.9, 1.0)>$	$<(0.0, 0.1.0.2, 0.3), (0.0, 0.1.0.2, 0.3)>$
N	$<(1.0, 1.0, 1.0, 1.0), (1.0, 1.0, 1.0, 1.0)>$	$<(0.0, 0.1.0.2, 0.3), (0.0, 0.1.0.2, 0.3)>$
	$<(0.1, 0.2, 0.3, 0.4), (0.0, 0.2, 0.3, 0.5)>$	$<(0.8, 0.9, 1.0, 1.0), (0.8, 0.9, 1.0, 1.0)>$

步骤1 7-2

7-3.

步骤2 7-17

$$E^1 = (0.075, 0.450, 0.650, 0.925); E^2 = (0.150, 0.250, 0.450, 1.000);$$
$$E^3 = (0.450, 0.450, 0.250, 0.250); E^4 = (0.450, 0.250, 0.075, 0.650);$$
$$E^5 = (0.450, 0.650, 0.250, 0.075); E^6 = (0.925, 0.075, 0.450, 0.250);$$
$$E^7 = (0.850, 0.150, 0.250, 0.925); E^8 = (0.650, 0.250, 0.450, 0.075);$$
$$E^9 = (0.250, 0.075, 0.250, 0.000).$$

$$E^P = (0.075, 0.925, 0.850, 0.150); \ E^N = (1.00, 0.150, 0.250, 0.925).$$

步骤3 7-21

$$S^{P1} = 0.5840; \ S^{P2} = 0.4308; \ S^{P3} = 0.5547; \ S^{P4} = 0.3035;$$
$$S^{P5} = 0.6114; \ S^{P6} = 0.3277; \ S^{P7} = 0.2777; \ S^{P8} = 0.4797;$$
$$S^{P9} = 0.4204;$$

$$S^{N1} = 0.5639; \ S^{N2} = 0.6298; \ S^{N3} = 0.5912; \ S^{N4} = 0.7263;$$
$$S^{N5} = 0.4455; \ S^{N6} = 0.6937; \ S^{N7} = 0.8321; \ S^{N8} = 0.5701;$$
$$S^{N9} = 0.4138.$$

步骤4 7-22

$$RS^1 = 1.0358; \ RS^2 = 0.6840; \ RS^3 = 0.9383; \ RS^4 = 0.4179; \ RS^5 = 1.3724;$$
$$RS^6 = 0.4724; \ RS^7 = 0.3337; \ RS^8 = 0.8414; \ RS^9 = 1.0161.$$

A_5,

7.2.4 小结

7.3 考虑属性二元关系的参数型直觉梯形模糊偏好多属性大群体决策

[162] [163]

[164] [165]

TOPSIS [166]

[167] [168]

[169]

[35]

Hamming

7.3.1 基于属性间二元关系的参数型直觉梯形模糊数的相似度

定义7.7 n

p m - k

i j $v_{ij}^k =<$
$(a_{ij1}^k; a_{ij2}^k; a_{ij3}^k; a_{ij4}^k); (b_{ij1}^k, b_{ij2}^k; b_{ij3}^k; b_{ij4}^k) >$ $b_{ij1}^k \cdot a_{ij1}^k \cdot b_{ij2}^k \cdot b_{ij2}^k \cdot a_{ij3}^k \cdot$
$b_{ij3}^k \cdot a_{ij4}^k \cdot b_{ij4}^k) >$ $V_i^k = (v_{i1}^k; v_{i2}^k; :::; v_{in}^k)$ k

i 7-17 k

i $E_i^k = (e_{i1}^k; e_{i2}^k; \ldots; e_{in}^k)$ $^{12}_{i} = ($

定义7.8 k i

$E_i^k = (e_{i1}^k; e_{i2}^k; \ldots; e_{in}^k)$ Q n E_i^k

$1 \cdot j_1; j_2 \cdot n$ $T_i^k(Q)$ k i

; :::;75 0 TD[<0a06>]TJ/F2 12.005 T0505565f9

Q $t^k_{j_1 j_2} = 1$ $(e^k_{ij_1}; e^k_{ij_2}) \in Q$ $t^k_{j_1 j_2} = 0$

$T_i^k(Q)$ 0-1

定义7.9 $E_i^1 = (e_{i1}^1; e_{i2}^1; \ldots; e_{in}^1)$ $E_i^2 = (e_{i1}^2; e_{i2}^2; \ldots; e_{in}^2)$ i

Q 7.8

$T_i^1(Q)$ $T_i^2(Q)$ i

V_i^1 V_i^2

$$S_i^{12}(V_i^1; V_i^2) = \frac{1}{\sqrt{n}} \frac{\|T_i^1 + T_i^2\|_2}{\|T_i^1\|_\infty + \|T_i^2\|_\infty}; \qquad (7\text{-}24)$$

$$\|T\|_2 = (\tfrac{1}{2}(T'T))^{\frac{1}{2}}; \quad \|T_i\|_\infty = \max_{1 \le j_1 \le n} \tfrac{1}{2}(T'T \ldots$$

步骤5 ª E^i $(e^i_j; e^i_l) \in Q$

Q $t^i_{jl} = 1$ $t^i_{jl} = 1$ T^i T^i

T ª E^i

步骤6 ª ª E^i $(i = i + 1)$

$E^i \in$ ª 5 ª 7

步骤7 $T = \{T^i\}$

7.3.1 Q 7-24

[88]

V^i V^j ◦ V^i

V^j $S(V^i; V^j)$ ¸

◦ V^i V^j ◦ ◦

$S(V^i; V^j)$

$\Omega = \{V^i | i = 1, 2, \ldots, m\}$ $T = \{T^i | i = 1, 2, \ldots, m\}$

[88] $K (1 \le K \le m)$ n_l l

$\sum_{l=1}^{K} n_l = m$ C^l

7.3.3 聚类案例分析

4 1

2 3 4 20 G

4 20

$\Omega = \{V_i | i = 1, 2, \ldots, 20\}$ 7-4

7-17

$$E^1 = (0.3975, 0.3800, 0.5243, 0.5732);\quad E^2 = (0.5364, 0.5090, 0.3232, 0.5617);$$
$$E^3 = (0.5357, 0.5236, 0.3815, 0.4907);\quad E^4 = (0.4807, 0.6170, 0.5691, 0.6265);$$
$$E^5 = (0.5620, 0.4769, 0.5944, 0.4346);\quad E^6 = (0.3673, 0.4469, 0.3959, 0.5792);$$
$$E^7 = (0.6965, 0.4169, 0.4874, 0.6065);\quad E^8 = (0.4687, 0.5166, 0.5863, 0.3979);$$
$$E^9 = (0.5426, 0.5009, 0.4187, 0.5696);\quad E^{10} = (0.5127, 0.3999, 0.6237, 0.3749);$$
$$E^{11} = (0.4567, 0.5327, 0.4967, 0.6143);\quad E^{12} = (0.3030, 0.3938, 0.6512, 0.5177);$$
$$E^{13} = (0.5156, 0.6112, 0.4988, 0.4183);\quad E^{14} = (0.6950, 0.5679, 0.5799, 0.4021);$$
$$E^{15} = (0.6709, 0.5610, 0.6421, 0.5079);\quad E^{16} = (0.5593, 0.5792, 0.5672, 0.5562);$$
$$E^{17} = (0.4885, 0.4941, 0.4807, 0.2552);\quad E^{18} = (0.3537, 0.4247, 0.4193, 0.7412);$$
$$E^{19} = (0.4086, 0.5737, 0.4689, 0.5231);\quad E^{20} = (0.5658, 0.4382, 0.5048, 0.5707).$$

表 7-4 20个决策专家的参数型直觉梯形模糊偏好矢量

	1 / 3	2 / 4
M1	$<(0.05,0.38,0.44,0.68),(0.00,0.24,0.52,0.85)>$	$<(0.08,0.28,0.47,0.59),(0.05,0.28,0.55,0.69)>$
	$<(0.11,0.55,0.62,0.89),(0.05,0.39,0.65,0.90)>$	$<(0.42,0.49,0.59,0.79),(0.33,0.45,0.63,0.87)>$
M2	$<(0.23,0.47,0.61,0.80),(0.04,0.35,0.79,0.97)>$	$<(0.10,0.48,0.69,0.90),(0.07,0.11,0.74,0.94)>$
	$<(0.08,0.25,0.31,0.56),(0.06,0.13,0.34,0.82)>$	$<(0.22,0.29,0.83,0.92),(0.13,0.27,0.84,0.94)>$
M3	$<(0.35,0.46,0.52,0.78),(0.25,0.44,0.65,0.80)>$	$<(0.09,0.43,0.58,0.91),(0.06,0.25,0.88,0.95)>$
	$<(0.13,0.27,0.43,0.57),(0.07,0.23,0.44,0.87)>$	$<(0.24,0.46,0.50,0.70),(0.20,0.28,0.70,0.81)>$
M4	$<(0.10,0.40,0.43,0.77),(0.09,0.40,0.74,0.88)>$	$<(0.18,0.74,0.75,0.85),(0.17,0.44,0.85,0.91)>$
	$<(0.38,0.57,0.69,0.71),(0.02,0.46,0.75,0.85)>$	$<(0.42,0.60,0.60,0.86),(0.29,0.58,0.73,0.89)>$
M5	$<(0.21,0.46,0.65,0.89),(0.15,0.37,0.78,0.95)>$	$<(0.22,0.51,0.51,0.57),(0.11,0.49,0.52,0.84)>$
	$<(0.21,0.48,0.78,0.97),(0.12,0.32,0.84,0.98)>$	$<(0.22,0.27,0.48,0.66),(0.17,0.27,0.52,0.84)>$
M6	$<(0.08,0.22,0.43,0.69),(0.05,0.11,0.62,0.70)>$	$<(0.14,0.41,0.46,0.74),(0.06,0.32,0.58,0.83)>$
	$<(0.03,0.07,0.32,0.88),(0.01,0.03,0.82,0.96)>$	$<(0.23,0.63,0.75,0.84),(0.08,0.29,0.82,0.96)>$
M7	$<(0.50,0.67,0.80,0.95),(0.23,0.57,0.86,0.97)>$	$<(0.13,0.34,0.52,0.57),(0.12,0.34,0.57,0.71)>$
	$<(0.09,0.46,0.60,0.85),(0.08,0.28,0.63,0.87)>$	$<(0.34,0.59,0.65,0.90),(0.19,0.51,0.69,0.93)>$
M8	$<(0.21,0.36,0.44,0.79),(0.13,0.29,0.60,0.88)>$	$<(0.39,0.44,0.54,0.75),(0.03,0.47,0.73,0.70)>$
	$<(0.44,0.52,0.55,0.78),(0.16,0.51,0.72,0.98)>$	$<(0.14,0.40,0.42,0.64),(0.00,0.27,0.46,0.85)>$
M9	$<(0.26,0.58,0.61,0.81),(0.10,0.27,0.77,0.89)>$	$<(0.26,0.45,0.48,0.75),(0.18,0.42,0.67,0.74)>$
	$<(0.09,0.42,0.44,0.89),(0.01,0.12,0.45,0.90)>$	$<(0.05,0.62,0.72,0.84),(0.02,0.55,0.82,0.90)>$
M10	$<(0.23,0.49,0.58,0.74),(0.19,0.40,0.63,0.80)>$	$<(0.17,0.27,0.30,0.70),(0.01,0.26,0.52,0.93)>$
	$<(0.33,0.68,0.72,0.87),(0.25,0.48,0.72,0.89)>$	$<(0.13,0.33,0.51,0.53),(0.05,0.28,0.53,0.60)>$
M11	$<(0.17,0.25,0.54,0.79),(0.12,0.21,0.58,0.95)>$	$<(0.26,0.43,0.45,0.91),(0.19,0.39,0.60,0.98)>$
	$<(0.15,0.25,0.69,0.76),(0.14,0.24,0.75,0.95)>$	$<(0.23,0.58,0.79,0.88),(0.20,0.42,0.85,0.92)>$
M12	$<(0.06,0.07,0.38,0.66),(0.01,0.06,0.45,0.70)>$	$<(0.18,0.33,0.37,0.60),(0.08,0.29,0.44,0.82)>$
	$<(0.43,0.72,0.73,0.95),(0.05,0.59,0.73,0.97)>$	$<(0.17,0.33,0.69,0.92),(0.09,0.20,0.74,0.97)>$
M13	$<(0.19,0.50,0.63,0.77),(0.03,0.49,0.64,0.83)>$	$<(0.37,0.61,0.65,0.86),(0.19,0.50,0.79,0.88)>$
	$<(0.26,0.42,0.42,0.82),(0.24,0.40,0.50,0.80)>$	$<(0.16,0.53,0.55,0.63),(0.00,0.17,0.61,0.65)>$
M14	$<(0.28,0.79,0.81,0.94),(0.12,0.75,0.85,0.98)>$	$<(0.23,0.55,0.67,0.87),(0.14,0.36,0.71,0.97)>$
	$<(0.04,0.60,0.67,0.93),(0.03,0.52,0.87,0.94)>$	$<(0.09,0.29,0.32,0.72),(0.01,0.11,0.68,0.95)>$
M15	$<(0.48,0.70,0.73,0.82),(0.35,0.51,0.77,0.97)>$	$<(0.38,0.59,0.60,0.63),(0.36,0.43,0.62,0.83)>$
	$<(0.37,0.56,0.58,0.92),(0.30,0.53,0.88,0.94)>$	$<(0.12,0.49,0.65,0.88),(0.05,0.19,0.66,0.97)>$
M16	$<(0.27,0.39,0.64,0.84),(0.26,0.39,0.72,0.91)>$	$<(0.13,0.44,0.65,0.98),(0.09,0.43,0.89,0.99)>$
	$<(0.15,0.40,0.77,0.91),(0.13,0.33,0.87,0.93)>$	$<(0.32,0.50,0.65,0.81),(0.01,0.47,0.75,0.89)>$
M17	$<(0.24,0.31,0.49,0.90),(0.00,0.29,0.68,0.97)>$	$<(0.17,0.41,0.61,0.89),(0.00,0.19,0.65,0.99)>$
	$<(0.11,0.38,0.58,0.85),(0.08,0.24,0.65,0.92)>$	$<(0.08,0.21,0.26,0.31),(0.06,0.10,0.29,0.69)>$
M18	$<(0.02,0.38,0.31,0.58),(0.02,0.03,0.40,0.97)>$	$<(0.03,0.42,0.52,0.63),(0.02,0.42,0.61,0.70)>$
	$<(0.14,0.34,0.47,0.65),(0.03,0.31,0.56,0.83)>$	$<(0.54,0.82,0.83,0.87),(0.33,0.69,0.86,0.96)>$
M19	$<(0.05,0.09,0.34,0.92),(0.04,0.08,0.74,0.97)>$	$<(0.40,0.54,0.55,0.81),(0.29,0.46,0.66,0.84)>$
	$<(0.24,0.33,0.52,0.84),(0.03,0.29,0.52,0.94)>$	$<(0.30,0.40,0.58,0.75),(0.25,0.32,0.69,0.85)>$
M20	$<(0.21,0.46,0.70,0.93),(0.10,0.34,0.80,0.90)>$	$<(0.13,0.54,0.54,0.60),(0.04,0.21,0.59,0.82)>$
	$<(0.05,0.50,0.67,0.78),(0.03,0.28,0.70,0.99)>$	$<(0.15,0.55,0.69,0.85),(0.02,0.52,0.79,0.95)>$

Q

1 Q_1 $(x_1 + x_2)$ ¸

$1{:}2;$ $(x_1; x_2)$ 2 Q_1

1.2

20 20 $T =$

$\{T^1; T^2; \dots; T^{20}\}$ 1 2 4

$$T^1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}; T^2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$T^4 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}.$$

$\gamma = 0.75$ $\Omega = \{V^i | i = 1, 2, \dots, m\}$ $T = \{T^i | i = 1, 2, \dots, m\}$ [88]

$K = 4$ 7-5

表 7-5 基于二元关系Q_1，$\gamma = 0.75$时的聚类算法执行结果

C^1	13	$E^1; E^4; E^5; E^6; E^7; E^8; E^9; E^{11}; E^{16}; E^{17}; E^{18}; E^{19}; E^{20}$
C_2	4	$E^2; E^{10}; E^{12}; E^{15}$
C_3	2	$E^3; E^{13}$
C_4	1	E^{14}

7-24 1 2 $S(V^1; V^2) = 0 < 0.75$ 2 1 4 $S(V^1; V^2) = 1 > 0.75$ 1 4 C^1

2 Q_2 $\log(x_1 + x_2)$ ¸ $0.1; (x_1; x_2) \in Q_2$

0.1 $\gamma = 0.75$ 7-6 , K=7

Q

Q

[88]

$\gamma = 0.75$ $\gamma = 0.9$ 7-4 [88]

7-7

表 7-6 基于二元关系Q_2，$\gamma = 0.75$时的聚类算法执行结果

(7-17) w_j $(j = 1, 2, \ldots, n)$

$$\hat{w}_j = \frac{EV(w_j)}{\sum_{i=1}^{n} EV(w_j)}. \tag{7-25}$$

步骤 1 7-2 m

7-17

步骤 2 A_1,

7.3.2

步骤 3 3-7 3-8 A_1

C_3 ;(4) C_4 12

步骤 1 7-2 12

7-8 7-9 7-10

表 7-8 12个决策专家关于方案A_1的参数型直觉梯形模糊偏好矢量

		1	2
		3	4
A_1	1	$<(0.0,0.0,0.1,0.2),(0.0,0.0,0.1,0.2)>$	$<(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)>$
		$<(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)>$	$<(0.8,0.9,1.0,1.0),(0.8,0.9,1.0,1.0)>$
	2	$<(0.0,0.1,0.2,0.3),(0.0,0.1,0.2,0.3)>$	$<(0.1,0.2,0.3,0.4),(0.0,0.2,0.3,0.5)>$
		$<(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)>$	$<(1.0,1.0,1.0,1.0),(1.0,1.0,1.0,1.0)>$
	3	$<(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)>$	$<(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)>$
		$<(0.1,0.2,0.3,0.4),(0.0,0.2,0.3,0.5)>$	$<(0.1,0.2,0.3,0.4),(0.0,0.2,0.3,0.5)>$
	4	$<(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)>$	$<(0.0,0.1,0.2,0.3),(0.0,0.1,0.2,0.3)>$
		$<(0.0,0.0,0.1,0.2),(0.0,0.0,0.1,0.2)>$	$<(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)>$
	5	$<(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)>$	$<(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)>$
		$<(0.1,0.2,0.3,0.4),(0.0,0.2,0.3,0.5)>$	$<(0.0,0.0,0.1,0.2),(0.0,0.0,0.1,0.2)>$
	6	$<(0.8,0.9,1.0,1.0),(0.8,0.9,1.0,1.0)>$	$<(0.0,0.0,0.1,0.2),(0.0,0.0,0.1,0.2)>$
		$<(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)>$	$<(0.1,0.2,0.3,0.4),(0.0,0.2,0.3,0.5)>$
	7	$<(0.7,0.8,0.9,1.0),(0.7,0.8,0.9,1.0)>$	$<(0.0,0.1,0.2,0.3),(0.0,0.1,0.2,0.3)>$
		$<(0.1,0.2,0.3,0.4),(0.0,0.2,0.3,0.5)>$	$<(0.8,0.9,1.0,1.0),(0.8,0.9,1.0,1.0)>$
	8	$<(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)>$	$<(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)>$
		$<(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)>$	$<(0.0,0.0,0.1,0.2),(0.0,0.0,0.1,0.2)>$
	9	$<(0.1,0.2,0.3,0.4),(0.0,0.2,0.3,0.5)>$	$<(0.0,0.0,0.1,0.2),(0.0,0.0,0.1,0.2)>$
		$<(0.1,0.2,0.3,0.4),(0.0,0.2,0.3,0.5)>$	$<(0.0,0.0,0.0,0.0),(0.0,0.0,0.0,0.0)>$
	10	$<(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)>$	$<(0.1,0.2,0.3,0.4),(0.0,0.2,0.3,0.5)>$
		$<(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)>$	$<(0.7,0.8,0.9,1.0),(0.7,0.8,0.9,1.0)>$
	11	$<(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)>$	$<(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)>$
		$<(0.0,0.1,0.2,0.3),(0.0,0.1,0.2,0.3)>$	$<(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)>$
	12	$<(0.7,0.8,0.9,1.0),(0.7,0.8,0.9,1.0)>$	$<(0.7,0.8,0.9,1.0),(0.7,0.8,0.9,1.0)>$
		$<(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)>$	$<(0.8,0.9,1.0,1.0),(0.8,0.9,1.0,1.0)>$

表 7-9 12个决策专家关于方案A_2的参数型直觉梯形模糊偏好矢量

		1	2
		3	4
A_2	1	$<(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)>$	$<(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)>$
		$<(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)>$	$<(0.0,0.0,0.1,0.2),(0.0,0.0,0.1,0.2)>$
	2	$<(0.0,0.1,0.2,0.3),(0.0,0.1,0.2,0.3)>$	$<(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)>$
		$<(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)>$	$<(0.8,0.9,1.0,1.0),(0.8,0.9,1.0,1.0)>$
	3	$<(0.7,0.8,0.9,1.0),(0.7,0.8,0.9,1.0)>$	$<(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)>$
		$<(0.1,0.2,0.3,0.4),(0.0,0.2,0.3,0.5)>$	$<(0.0,0.1,0.2,0.3),(0.0,0.1,0.2,0.3)>$
	4	$<(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)>$	$<(0.7,0.8,0.9,1.0),(0.7,0.8,0.9,1.0)>$
		$<(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)>$	$<(1.0,1.0,1.0,1.0),(1.0,1.0,1.0,1.0)>$
	5	$<(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)>$	$<(0.7,0.8,0.9,1.0),(0.7,0.8,0.9,1.0)>$
		$<(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)>$	$<(0.8,0.9,1.0,1.0),(0.8,0.9,1.0,1.0)>$
	6	$<(0.7,0.8,0.9,1.0),(0.7,0.8,0.9,1.0)>$	$<(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)>$
		$<(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)>$	$<(0.1,0.2,0.3,0.4),(0.0,0.2,0.3,0.5)>$
	7	$<(0.0,0.1,0.2,0.3),(0.0,0.1,0.2,0.3)>$	$<(0.7,0.8,0.9,1.0),(0.7,0.8,0.9,1.0)>$
		$<(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)>$	$<(0.1,0.2,0.3,0.4),(0.0,0.2,0.3,0.5)>$
	8	$<(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)>$	$<(0.1,0.2,0.3,0.4),(0.0,0.2,0.3,0.5)>$
		$<(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)>$	$<(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)>$
	9	$<(0.7,0.8,0.9,1.0),(0.7,0.8,0.9,1.0)>$	$<(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)>$
		$<(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)>$	$<(0.0,0.1,0.2,0.3),(0.0,0.1,0.2,0.3)>$
	10	$<(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)>$	$<(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)>$
		$<(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)>$	$<(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)>$
	11	$<(0.0,0.1,0.2,0.3),(0.0,0.1,0.2,0.3)>$	$<(0.1,0.2,0.3,0.4),(0.0,0.2,0.3,0.5)>$
		$<(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)>$	$<(0.8,0.9,1.0,1.0),(0.8,0.9,1.0,1.0)>$
	12	$<(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)>$	$<(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)>$
		$<(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)>$	$<(0.1,0.2,0.3,0.4),(0.0,0.2,0.3,0.5)>$

7-17 12

$E_1^1 = (0{:}075; 0{:}450; 0{:}650; 0{:}925)$; $E_1^2 = (0{:}150; 0{:}250; 0{:}450; 1{:}000)$;

$E_1^3 = (0{:}450; 0{:}450; 0{:}250; 0{:}250)$; $E_1^4 = (0{:}450; 0{:}250; 0{:}075; 0{:}650)$;

$E_1^5 = (0{:}450; 0{:}650; 0{:}250; 0{:}075)$; $E_1^6 = (0{:}925; 0{:}075; 0{:}450; 0{:}250)$;

$E_1^7 = (0{:}850; 0{:}150; 0{:}250; 0{:}925)$; $E_1^8 = (0{:}650; 0{:}250; 0{:}450; 0{:}075)$;

$E_1^9 = (0{:}250; 0{:}075; 0{:}250; 0{:}000)$; $E_1^{10} = (0{:}650; 0{:}250; 0{:}650; 0{:}850)$;

$E_1^{11} = (0{:}450; 0{:}650; 0{:}150; 0{:}450)$; $E_1^{12} = (0{:}850; 1{:}000; 0{:}450; 0{:}925)$;

$E_2^1 = (0{:}450; 0{:}650; 0{:}450; 0{:}075)$; $E_2^2 = (0{:}150; 0{:}450; 0{:}650; 0{:}925)$;

$E_2^3 = (0{:}850; 0{:}650; 0{:}250; 0{:}150)$; $E_2^4 = (0{:}650; 0{:}850; 0{:}450; 1{:}000)$;

$E_2^5 = (0{:}650; 0{:}850; 0{:}450; 0{:}850)$; $E_2^6 = (0{:}850; 0{:}650; 0{:}450; 0{:}250)$;

$E_2^7 = (0{:}150; 0{:}850; 0{:}450; 0{:}250)$; $E_2^8 = (0{:}650; 0{:}850; 0{:}450; 0{:}650)$;

$E_2^9 = (0{:}850; 0{:}650; 0{:}450; 0{:}150)$; $E_2^{10} = (0{:}650; 0{:}450; 0{:}650; 0{:}450)$;

表 7-10 12个决策专家关于方案A_3的参数型直觉梯形模糊偏好矢量

		1	2
		3	4
A_3	1	$<(0.1,0.2,0.3,0.4),(0.0,0.2,0.3,0.5)>$	$<(0.0,0.1,0.2,0.3),(0.0,0.1,0.2,0.3)>$
		$<(0.7,0.8,0.9,1.0),(0.7,0.8,0.9,1.0)>$	$<(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)>$
	2	$<(0.0,0.1,0.2,0.3),(0.0,0.1,0.2,0.3)>$	$<(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)>$
		$<(0.1,0.2,0.3,0.4),(0.0,0.2,0.3,0.5)>$	$<(0.0,0.0,0.0,0.0),(0.0,0.0,0.0,0.0)>$
	3	$<(0.8,0.9,1.0,1.0),(0.8,0.9,1.0,1.0)>$	$<(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)>$
		$<(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)>$	$<(1.0,1.0,1.0,1.0),(1.0,1.0,1.0,1.0)>$
	4	$<(0.0,0.1,0.2,0.3),(0.0,0.1,0.2,0.3)>$	$<(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)>$
		$<(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)>$	$<(0.0,0.0,0.1,0.2),(0.0,0.0,0.1,0.2)>$
	5	$<(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)>$	$<(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)>$
		$<(0.1,0.2,0.3,0.4),(0.0,0.2,0.3,0.5)>$	$<(0.0,0.0,0.1,0.2),(0.0,0.0,0.1,0.2)>$
	6	$<(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)>$	$<(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)>$
		$<(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)>$	$<(0.1,0.2,0.3,0.4),(0.0,0.2,0.3,0.5)>$
	7	$<(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)>$	$<(0.1,0.2,0.3,0.4),(0.0,0.2,0.3,0.5)>$
		$<(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)>$	$<(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)>$
	8	$<(0.1,0.2,0.3,0.4),(0.0,0.2,0.3,0.5)>$	$<(0.1,0.2,0.3,0.4),(0.0,0.2,0.3,0.5)>$
		$<(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)>$	$<(0.7,0.8,0.9,1.0),(0.7,0.8,0.9,1.0)>$
	9	$<(0.0,0.1,0.2,0.3),(0.0,0.1,0.2,0.3)>$	$<(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)>$
		$<(0.1,0.2,0.3,0.4),(0.0,0.2,0.3,0.5)>$	$<(0.1,0.2,0.3,0.4),(0.0,0.2,0.3,0.5)>$
	10	$<(0.7,0.8,0.9,1.0),(0.7,0.8,0.9,1.0)>$	$<(0.7,0.8,0.9,1.0),(0.7,0.8,0.9,1.0)>$
		$<(0.1,0.2,0.3,0.4),(0.0,0.2,0.3,0.5)>$	$<(0.1,0.2,0.3,0.4),(0.0,0.2,0.3,0.5)>$
	11	$<(0.0,0.1,0.2,0.3),(0.0,0.1,0.2,0.3)>$	$<(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)>$
		$<(0.0,0.0,0.1,0.2),(0.0,0.0,0.1,0.2)>$	$<(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)>$
	12	$<(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)>$	$<(0.7,0.8,0.9,1.0),(0.7,0.8,0.9,1.0)>$
		$<(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)>$	$<(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)>$

$E_2^{11} = (0.450, 0.650, 0.150, 0.450)$; $E_2^{12} = (0.450, 0.650, 0.450, 0.250)$;

$E_3^{1} = (0.250, 0.150, 0.850, 0.650)$; $E_3^{2} = (0.150, 0.650, 0.250, 0.000)$;

$E_3^{3} = (0.925, 0.450, 0.650, 1.000)$; $E_3^{4} = (0.150, 0.650, 0.450, 0.075)$;

$E_3^{5} = (0.450, 0.450, 0.250, 0.075)$; $E_3^{6} = (0.650, 0.650, 0.450, 0.250)$;

$E_3^{7} = (0.450, 0.250, 0.650, 0.450)$; $E_3^{8} = (0.250, 0.250, 0.650, 0.850)$;

$E_3^{9} = (0.150, 0.650, 0.250, 0.250)$; $E_3^{10} = (0.850, 0.850, 0.250, 0.250)$;

$E_3^{11} = (0.150, 0.650, 0.075, 0.650)$; $E_3^{12} = (0.450, 0.850, 0.450, 0.450)$.

步骤 2-3

$\log(x_1 + x_2)$ ¸ 0.4 $\pm = 0.75$

(7-24) 7.3.2 7-11

步骤 4 7-11 :

表 7-11 当$\delta = 0.75$，各个方案的聚类结果

		r		
A_1	8	1, 2, 3, 4, 5, 8, 9, 11	(0.4888, 0.5055, 0.4219, 0.5723)	(0.5429, 0.4348,
	3	3, 6, 7, 12	(0.6985, 0.3260, 0.3060, 0.5588)	0.4002, 0.5767)
	1	10	(0.5091, 0.1958, 0.5091, 0.6658)	
A_2	5	1, 2, 10, 11, 12	(0.3742, 0.4956, 0.5765, 0.5310)	(0.4805, 0.5799,
	7	3, 4, 5, 6, 7, 8, 9	(0.5564, 0.6402, 0.3530, 0.3949)	0.4461, 0.4516)
A_3	1	1	(0.2254, 0.1353, 0.7664, 0.5861)	
	8	2, 4, 5, 6, 7, 8, 9, 11	(0.3828, 0.6699, 0.4825, 0.4147)	(0.4148, 0.6018,
	1	3	(0.5873, 0.2857, 0.4127, 0.6349)	0.4694, 0.4277)
	2	10, 12	(0.5513, 0.7210, 0.2969, 0.2969)	

$$T^U = \begin{pmatrix} 0.5429 & 0.4805 & 0.4148 \\ 0.4348 & 0.5799 & 0.6018 \\ 0.4002 & 0.4461 & 0.4694 \\ 0.5767 & 0.4516 & 0.4277 \end{pmatrix}.$$

步骤 5 C_i

$$w_1 = < C_1, (0.3, 0.4, 0.5, 0.6), (0.2, 0.4, 0.5, 0.7) >,$$
$$w_2 = < C_2, (0.7, 0.8, 0.9, 1.0), (0.7, 0.8, 0.9, 1.0) >,$$
$$w_3 = < C_3, (0.1, 0.2, 0.3, 0.4), (0.0, 0.2, 0.3, 0.5) >,$$
$$w_4 = < C_4, (0.8, 0.9, 1.0, 1.0), (0.8, 0.9, 1.0, 1.0) > .$$

7-17 7-25

$$\hat{W} = (0.2430, 0.3178, 0.0935, 0.3458).$$

步骤 6 7-27

$$O = \hat{W} T^U = (0.2430, 0.3178, 0.0935, 0.3458) \begin{pmatrix} 0.5429 & 0.4805 & 0.4148 \\ 0.4348 & 0.5799 & 0.6018 \\ 0.4002 & 0.4461 & 0.4694 \\ 0.5767 & 0.4516 & 0.4277 \end{pmatrix}$$

$$= (0.5070, 0.4989, 0.4838).$$

$A_1 \succ A_2 \succ A_3$

A_1

7.3.6 小结

7.4 区间梯形模糊数型偏好多属性大群体决策方法

Gorzalczany[170] Turksen[171]
Wang Li
[172]
[173] [174]
Balezentis Zeng [175] MULTIMOORA(Multi-Objective Optimization by Ratio Analysis plus the Full Multiplicative From)
MULTIMOORA [176]
[177] Ashtiani
TOPSIS
Wei Chen [178]
Liu [38] [179]

7.4.1 区间梯形模糊数及相似度

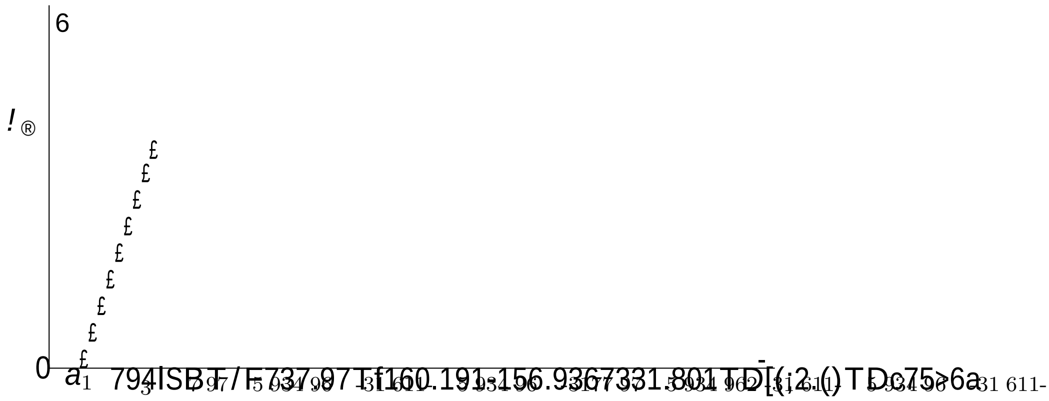
7.1

G-

定义7.10 G- [180] ® = $(a_1; a_2; a_3; a_4; !_®)$(7-3)

$¹_®(x) : R\ \ !\ \ [0; 1]$

$$
¹_®(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} \ £\ !_®; & x\ 2\ (a_1; a_2); \\ !_®; & x\ 2\ (a_2; a_3); \\ \frac{x-a_4}{a_3-a_4} \ £\ !_®; & x\ 2\ (a_3; a_4); \\ 0; & x\ 2\ (\ ¡\ 1\ ; a_1)\ [\ (a_4; +1\ ): \end{cases} \quad (7\text{-}29)
$$



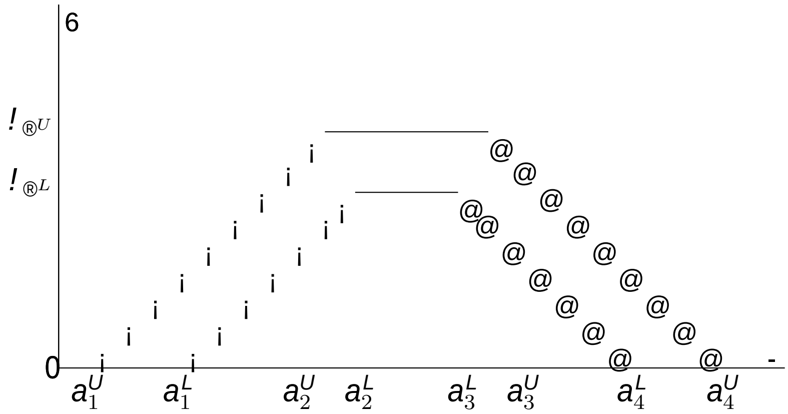


图 7-4 区间梯形模糊数的图像

$b_1^L \cdot b_2^L \cdot b_3^L \cdot b_4^L \cdot 1 \quad 0 \cdot b_1^U \cdot b_2^U \cdot b_3^U \cdot b_4^U \cdot 1 \quad 0 \cdot !_{¯^L} \cdot !_{¯^U} \cdot 1$

$®^L \ \mu \ ®^U \quad ¯^L \ \mu \ ¯^U$ [178]

1

$$
\begin{aligned}
® © ¯ &= [(a_1^L; a_2^L; a_3^L; a_4^L; !_{®^L}); (a_1^U; a_2^U; a_3^U; a_4^U; !_{®^U})] \\
&\quad ©[(b_1^L; b_2^L; b_3^L; b_4^L; !_{¯^L}); (b_1^U; b_2^U; b_3^U; b_4^U; !_{¯^U})] \\
&= [(a_1^L + b_1^L; a_2^L + b_2^L; a_3^L + b_3^L; a_4^L + b_4^L; min(!_{®^L}; !_{¯^L}); \\
&\quad (a_1^U + b_1^U; a_2^U + b_2^U; a_3^U + b_3^U; a_4^U + b_4^U; min(!_{®^U}; !_{¯^U})];
\end{aligned}
$$

2

$$
\begin{aligned}
® ª ¯ &= [(a_1^L; a_2^L; a_3^L; a_4^L; !_{®^L}); (a_1^U; a_2^U; a_3^U; a_4^U; !_{®^U})] \\
&\quad ª[(b_1^L; b_2^L; b_3^L; b_4^L; !_{¯^L}); (b_1^U; b_2^U; b_3^U; b_4^U; !_{¯^U})] \\
&= [(a_1^L ¡ b_1^L; a_2^L ¡ b_2^L; a_3^L ¡ b_3^L; a_4^L ¡ b_4^L; min(!_{®^L}; !_{¯^L}); \\
&\quad (a_1^U ¡ b_1^U; a_2^U ¡ b_2^U; a_3^U ¡ b_3^U; a_4^U ¡ b_4^U; min(!_{®^U}; !_{¯^U})];
\end{aligned}
$$

3

$$
\begin{aligned}
® - ¯ &= [(a_1^L; a_2^L; a_3^L; a_4^L; !_{®^L}); (a_1^U; a_2^U; a_3^U; a_4^U; !_{®^U})] \\
&\quad -[(b_1^L; b_2^L; b_3^L; b_4^L; !_{¯^L}); (b_1^U; b_2^U; b_3^U; b_4^U; !_{¯^U})] \\
&= [(a_1^L £ b_1^L; a_2^L £ b_2^L; a_3^L £ b_3^L; a_4^L £ b_4^L; min(!_{®^L}; !_{¯^L}); \\
&\quad (a_1^U £ b_1^U; a_2^U £ b_2^U; a_3^U £ b_3^U; a_4^U £ b_4^U; min(!_{®^U}; !_{¯^U})];
\end{aligned}
$$

4

$$
\begin{aligned}
® ® ¯ &= [(a_1^L; a_2^L; a_3^L; a_4^L; !_{®^L}); (a_1^U; a_2^U; a_3^U; a_4^U; !_{®^U})] \\
&\quad ®[(b_1^L; b_2^L; b_3^L; b_4^L; !_{¯^L}); (b_1^U; b_2^U; b_3^U; b_4^U; !_{¯^U})] \\
&= [(a_1^L/b_1^L; a_2^L/b_2^L; a_3^L/b_3^L; a_4^L/b_4^L; min(!_{®^L}; !_{¯^L}); \\
&\quad (a_1^U/b_1^U; a_2^U/b_2^U; a_3^U/b_3^U; a_4^U/b_4^U; min(!_{®^U}; !_{¯^U})];
\end{aligned}
$$

5

$$¸® = ¸ £ [(¸a^L_1; ¸a^L_2; ¸a^L_3; ¸a^L_4; !_{®^L}); (¸a^U_1; ¸a^U_2; ¸a^U_3; ¸a^U_4; !_{®^U})]:$$

Liu [38]

$$® = [®^L; ®^U] = [(a^L_1 \quad a^L_2; a^L_3; a^L_4; !_{®^L}); (a^U_1; a^U_2; a^U_3; a^U_4; !_{®^U})] \quad ¯$$

$$= [¯^L; ¯^U] = [(b^L_1 \quad b^L_2; b^L_3; b^L_4; !_{¯^L}); (b^U_1; b^U_2; b^U_3; b^U_4; !_{¯^U})]$$

G- 7-30 $®^L; ®^U; ¯^L \quad ¯^U$

$(X_{®^L}; Y_{®^L})$ $(X_{®^U}; Y_{®^U})$ $(X_{¯^L}; Y_{¯^L})$ $(X_{¯^U}; Y_{¯^U})$

® ¯

$$\begin{cases} X_® = (X_{®^L} + X_{®^U})=2; \\ Y_® = (Y_{®^L} + Y_{®^U})=2; \end{cases} \tag{7-31}$$

$$\begin{cases} X_¯ = (X_{¯^L} + X_{¯^U})=2; \\ Y_¯ = (Y_{¯^L} + Y_{¯^U})=2; \end{cases} \tag{7-32}$$

$$d_® = \sqrt{X^2_® + Y^2_®}; \tag{7-33}$$

$$d_¯ = \sqrt{X^2_¯ + Y^2_¯}: \tag{7-34}$$

$d_®$ ¸ $d_¯$ ® ¸ ¯ ® ¯

定义7.11 $® = [®^L; ®^U] = [(a^L_1, a^L_2; a^L_3; a^L_4; !_{®^L}); (a^U_1; a^U_2; a^U_3; a^U_4; !_{®^U})]$ ¯ $= [¯^L; ¯^U] = [(b^L_1, b^L_2; b^L_3; b^L_4; !_{¯^L}); (b^U_1; b^U_2; b^U_3; b^U_4; !_{¯^U})]$

$$S(®;¯) = 1 ¡ min(!_{®^L}; !_{¯^L}) £ \frac{\sum_{i=1}^{4} ja^L_i ¡ b^L_i j}{4}$$
$$¡ min(!_{®^U}; !_{¯^U}) £ \frac{\sum_{i=1}^{4} ja^U_i ¡ b^U_i j}{4}: \tag{7-35}$$

定理 7.5. 定义*7.11*中两个区间梯形模糊数之间的相似度$S(®;¯)$, 满足相似度的性质。

证明. P1 $S(®;¯) ¸ 0$ $S(®;¯) · 1$

$0 · a^L_1 · a^L_2 · a^L_3 · a^L_4 · 1$ $0 · a^U_1 · a^U_2 · a^U_3 · a^U_4 · 1$ $0 · !_{®^L} · !_{®^U} · 1$ $0 · b^L_1 · b^L_2 · b^L_3 · b^L_4 · 1$ $0 · b^U_1 · b^U_2 · b^U_3 · b^U_4 · 1$ $0 · !_{¯^L} · !_{¯^U} · 1$

$$min(!_{®^L}; !_{¯^L}) £ \frac{\sum_{i=1}^{4} ja^L_i ¡ b^L_i j}{4} + min(!_{®^U}; !_{¯^U}) £ \frac{\sum_{i=1}^{4} ja^U_i ¡ b^U_i j}{4} ¸ 0; \tag{7-36}$$

7-36 7-35 $S(\alpha,\beta) \le 1$ $0 \le S(\alpha,\beta) \le 1$

P2 $\alpha = \beta$ $a_1^L = b_1^L, a_2^L = b_2^L, a_3^L = b_3^L, a_4^L = b_4^L$ $a_1^U = b_1^U, a_2^U = b_2^U, a_3^U = b_3^U, a_4^U = b_4^U$ 7-35

$$S(\alpha,\beta) = 1.$$

(P3)

$$\begin{aligned} S(\alpha,\beta) &= 1 - \min(\omega_{\alpha^L}, \omega_{\beta^L}) \times \frac{\sum_{i=1} |a_i^L - b_i^L|}{4} \\ &\quad - \min(\omega_{\alpha^U}, \omega_{\beta^U}) \times \frac{\sum_{i=1} |a_i^U - b_i^U|}{4} = S(\beta,\alpha). \end{aligned}$$

□

7.4.2 区间梯形模糊数型偏好多属性大群体决策方法

p $A = \{A_1, A_2, \ldots, A_p\}$,

n $(C = \{C_1, C_2, \ldots, C_n\})$ p m

m $G = \{G_1, G_2, \ldots, G_m\}$

n k G_k i

A_i

$$\begin{aligned} v_i^k = \{ &< C_1, [(a_{i11}^{kL}, a_{i12}^{kL}, a_{i13}^{kL}, a_{i14}^{kL}; \omega^k_{C_1^L}), (b_{i11}^{kU}, b_{i12}^{kU}, b_{i13}^{kU}, b_{i14}^{kU}; \omega^k_{C_1^U})] >, \\ &< C_2, [(a_{i21}^{kL}, a_{i22}^{kL}, a_{i23}^{kL}, a_{i24}^{kL}; \omega^k_{C_2^L}), (b_{i21}^{kU}, b_{i22}^{kU}, b_{i23}^{kU}, b_{i24}^{kU}; \omega^k_{C_2^U})] >, \ldots, \\ &< C_n, [(a_{in1}^{kL}, a_{in2}^{kL}, a_{in30160}^{kL} \end{aligned}$$

24; !L); /F212.813d4156L 2 [0 35] 212 813 4156

A_i

$$
\begin{aligned}
U_i = \{ &\langle C_1, [(\frac{\sum_{k=1} a_{i11}^{kL}}{m}, \frac{\sum_{k=1} a_{i12}^{kL}}{m}, \frac{\sum_{k=1} a_{i13}^{kL}}{m}, \frac{\sum_{k=1} a_{i14}^{kL}}{m}; \min\{\omega_{C_1^L}^{k}\}), \\
&(\frac{\sum_{k=1} b_{i11}^{kU}}{m}, \frac{\sum_{k=1} b_{i12}^{kU}}{m}, \frac{\sum_{k=1} b_{i13}^{kU}}{m}, \frac{\sum_{k=1} b_{i14}^{kU}}{m}; \min\{\omega_{C_1^U}^{k}\})]\rangle, \\
&\langle C_2, [(\frac{\sum_{k=1} a_{i21}^{kL}}{m}, \frac{\sum_{k=1} a_{i22}^{kL}}{m}, \frac{\sum_{k=1} a_{i23}^{kL}}{m}, \frac{\sum_{k=1} a_{i24}^{kL}}{m}; \min\{\omega_{C_2^L}^{k}\}), \\
&(\frac{\sum_{k=1} b_{i21}^{kU}}{m}, \frac{\sum_{k=1} b_{i22}^{kU}}{m}, \frac{\sum_{k=1} b_{i23}^{kU}}{m}, \frac{\sum_{k=1} b_{i24}^{kU}}{m}; \min\{\omega_{C_2^U}^{k}\})]\rangle, \ldots, \\
&\langle C_n, [(\frac{\sum_{k=1} a_{in1}^{kL}}{m}, \frac{\sum_{k=1} a_{in2}^{kL}}{m}, \frac{\sum_{k=1} a_{in3}^{kL}}{m}, \frac{\sum_{k=1} a_{in4}^{kL}}{m}; \min\{\omega_{C_n^L}^{k}\}), \\
&(\frac{\sum_{k=1} b_{in1}^{kU}}{m}, \frac{\sum_{k=1} b_{in2}^{kU}}{m}, \frac{\sum_{k=1} b_{in3}^{kU}}{m}, \frac{\sum_{k=1} b_{in4}^{kU}}{m}; \min\{\omega_{C_n^U}^{k}\})]\rangle, \qquad (7\text{-}37)
\end{aligned}
$$

C_j $(j = 1, 2, \ldots, n)$

C_j $w_j = [(a_1^L, a_2^L, a_3^L, a_4^L; \omega_{w_j^L}), (a_1^U, a_2^U, a_3^U, a_4^U; \omega_{w_j^U})]$ $(j = 1, 2, \ldots, n)$ 7-33

w_j $(j = 1, 2, \ldots, n)$

(7-33) w_j $(j = 1, 2, \ldots, n)$

$$\hat{w}_j = \frac{d(w_j)}{\sum_{i=1} d(w_j)}. \qquad (7\text{-}38)$$

步骤 1 7-12 m

步骤 2 7-37 p

步骤 3

步骤 4 7-35

$$T = \begin{pmatrix} t_1^1 & t_2^1 & \cdots & t_p^1 \\ t_1^2 & t_2^2 & \cdots & t_p^2 \\ \vdots & \vdots & \vdots & \vdots \\ t_1^n & t_2^n & \cdots & t_p^n \end{pmatrix},$$

表 7-12 语言值与区间梯形模糊数对照表

1	$[(0.00, 0.00, 0.00, 0.00; 0.80), (0.00, 0.00, 0.00, 0.00; 1.00)]$
2	$[(0.00, 0.00, 0.02, 0.07; 0.80), (0.00, 0.00, 0.02, 0.07; 1.00)]$
3	$[(0.04, 0.10, 0.18, 0.23; 0.80), (0.04, 0.10, 0.18, 0.23; 1.00)]$
4	$[(0.17, 0.22, 0.36, 0.42; 0.80), (0.17, 0.22, 0.36, 0.42; 1.00)]$
5	$[(0.32, 0.41, 0.58, 0.65; 0.80), (0.32, 0.41, 0.58, 0.65; 1.00)]$
6	$[(0.58, 0.63, 0.80, 0.86; 0.80), (0.58, 0.63, 0.80, 0.86; 1.00)]$
7	$[(0.72, 0.78, 0.92, 0.97; 0.80), (0.72, 0.78, 0.92, 0.97; 1.00)]$
8	$[(0.93, 0.98, 1.00, 1.00; 0.80), (0.93, 0.98, 1.00, 1.00; 1.00)]$
9	$[(1.00, 1.00, 1.00, 1.00; 0.80), (1.00, 1.00, 1.00, 1.00; 1.00)]$

t_i^j i j j

步骤 5 7-33 7-38

$$\hat{W} = (\hat{w}_1, \hat{w}_2, \ldots, \hat{w}_n).$$

步骤 6 $A = \{A_1, A_2, \ldots, A_p\}$

$$O = \hat{W}T = (\hat{w}_1, \hat{w}_2, \ldots, \hat{w}_n)\begin{pmatrix} t_1^1 & t_2^1 & \cdots & t_p^1 \\ t_1^2 & t_2^2 & \cdots & t_p^2 \\ \vdots & \vdots & \vdots & \vdots \\ t_1^n & t_2^n & \cdots & t_p^n \end{pmatrix} = (O_1, O_2, \ldots, O_p). \qquad (7\text{-}39)$$

$$Best\ alternative = \max_{1 \le i \le p} \{O_i\}. \qquad (7\text{-}40)$$

7.4.3 案例分析

10 4 4 1(A_1) 5 1 2(A_2) 2 3(A_3) 4(A_4) 10 1 (C_1) 2 (C_2) 3 (C_3) 4 (C_4) [4])

Ã Ã

表 7-13 10个决策专家关于方案A_1和A_2的区间梯形模糊偏好矢量

		1 3	2 4
A_1	1	[(0.00, 0.00, 0.02, 0.07; 0.8), (0.00, 0.00, 0.02, 0.07; 1.0)]	[(0.32, 0.41, 0.58, 0.65; 0.8), (0.32, 0.41, 0.58, 0.65; 1.0)]
		[(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.0)]	[(0.93, 0.98, 1.00, 1.00; 0.8), (0.93, 0.98, 1.00, 1.00; 1.0)]
	2	[(0.04, 0.10, 0.18, 0.23; 0.8), (0.04, 0.10.0.18, 0.23; 1.0)]	[(0.17, 0.22, 0.36, 0.42; 0.8), (0.17, 0.22, 0.36, 0.42; 1.0)]
		[(0.32, 0.41, 0.58, 0.65; 0.8), (0.32, 0.41, 0.58, 0.65; 1.0)]	[(1.00, 1.00, 1.00, 1.00; 0.8), (1.00, 1.00, 1.00, 1.00; 1.0)]
	3	[(0.32, 0.41, 0.58, 0.65; 0.8), (0.32, 0.41, 0.58, 0.65; 1.0)]	[(0.32, 0.41, 0.58, 0.65; 0.8), (0.32, 0.41, 0.58, 0.65; 1.0)]
		[(0.17, 0.22, 0.36, 0.42; 0.8), (0.17, 0.22, 0.36, 0.42; 1.0)]	[(0.17, 0.22, 0.36, 0.42; 0.8), (0.17, 0.22, 0.36, 0.42; 1.0)]
	4	[(0.32, 0.41, 0.58, 0.65; 0.8), (0.32, 0.41, 0.58, 0.65; 1.0)]	[(0.04, 0.10, 0.18, 0.23; 0.8), (0.04, 0.10, 0.18, 0.23; 1.0)]
		[(0.00, 0.00, 0.02, 0.07; 0.8), (0.00, 0.00, 0.02, 0.07; 1.0)]	[(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.0)]
	5	[(0.32, 0.41, 0.58, 0.65; 0.8), (0.32, 0.41, 0.58, 0.65; 1.0)]	[(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.0)]
		[(0.17, 0.22, 0.36, 0.42; 0.8), (0.17, 0.22, 0.36, 0.42; 1.0)]	[(0.00, 0.00, 0.02, 0.07; 0.8), (0.00, 0.00, 0.02, 0.07; 1.0)]
	6	[(0.93, 0.98, 1.00, 1.00; 0.8), (0.93, 0.98, 1.00, 1.00; 1.0)]	[(0.00, 0.00, 0.02, 0.07; 0.8), (0.00, 0.00, 0.02, 0.07; 1.0)]
		[(0.32, 0.41, 0.58, 0.65; 0.8), (0.32, 0.41, 0.58, 0.65; 1.0)]	[(0.17, 0.22, 0.36, 0.42; 0.8), (0.17, 0.22, 0.36, 0.42; 1.0)]
	7	[(0.72, 0.78, 0.92, 0.97; 0.8), (0.72, 0.78, 0.92, 0.97; 1.0)]	[(0.04, 0.10, 0.18, 0.23; 0.8), (0.04, 0.10, 0.18, 0.23; 1.0)]
		[(0.17, 0.22, 0.36, 0.42; 0.8), (0.17, 0.22, 0.36, 0.42; 1.0)]	[(0.93, 0.98, 1.00, 1.00; 0.8), (0.93, 0.98, 1.00, 1.00; 1.0)]
	8	[(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.0)]	[(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.0)]
		[(0.32, 0.41, 0.58, 0.65; 0.8), (0.32, 0.41, 0.58, 0.65; 1.0)]	[(0.00, 0.00, 0.02, 0.07; 0.8), (0.00, 0.00, 0.02, 0.07; 1.0)]
	9	[(0.17, 0.22, 0.36, 0.42; 0.8), (0.17, 0.22, 0.36, 0.42; 1.0)]	[(0.00, 0.00, 0.02, 0.07; 0.8), (0.00, 0.00, 0.02, 0.07; 1.0)]
		[(0.17, 0.22, 0.36, 0.42; 0.8), (0.17, 0.22, 0.36, 0.42; 1.0)]	[(0.00, 0.00, 0.00, 0.00; 0.8), (0.00, 0.00, 0.00, 0.00; 1.0)]
	10	[(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.0)]	[(0.17, 0.22, 0.36, 0.42; 0.8), (0.17, 0.22, 0.36, 0.42; 1.0)]
		[(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.0)]	[(0.72, 0.78, 0.92, 0.97; 0.8), (0.72, 0.78, 0.92, 0.97; 1.0)]
A_2	1	[(0.32, 0.41, 0.58, 0.65; 0.8), (0.32, 0.41, 0.58, 0.65; 1.0)]	[(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.0)]
		[(0.32, 0.41, 0.58, 0.65; 0.8), (0.32, 0.41, 0.58, 0.65; 1.0)]	[(0.00, 0.00, 0.02, 0.07; 0.8), (0.00, 0.00, 0.02, 0.07; 1.0)]
	2	[(0.04, 0.10.0.18, 0.23; 0.8), (0.04, 0.10.0.18, 0.23; 1.0)]	[(0.32, 0.41, 0.58, 0.65; 0.8), (0.32, 0.41, 0.58, 0.65; 1.0)]
		[(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.0)]	[(0.93, 0.98, 1.00, 1.00; 0.8), (0.93, 0.98, 1.00, 1.00; 1.0)]
	3	[(0.72, 0.78, 0.92, 0.97; 0.8), (0.72, 0.78, 0.92, 0.97; 1.0)]	[(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.0)]
		[(0.17, 0.22, 0.36, 0.42; 0.8), (0.17, 0.22, 0.36, 0.42; 1.0)]	[(0.04, 0.10, 0.18, 0.23; 0.8), (0.04, 0.10, 0.18, 0.23; 1.0)]
	4	[(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.0)]	[(0.72, 0.78, 0.92, 0.97; 0.8), (0.72, 0.78, 0.92, 0.97; 1.0)]
		[(0.32, 0.41, 0.58, 0.65; 0.8), (0.32, 0.41, 0.58, 0.65; 1.0)]	[(1.00, 1.00, 1.00, 1.00; 0.8), (1.00, 1.00, 1.00, 1.00; 1.0)]
	5	[(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.0)]	[(0.72, 0.78, 0.92, 0.97; 0.8), (0.72, 0.78, 0.92, 0.97; 1.0)]
		[(0.32, 0.41, 0.58, 0.65; 0.8), (0.32, 0.41, 0.58, 0.65; 1.0)]	[(0.93, 0.98, 1.00, 1.00; 0.8), (0.93, 0.98, 1.00, 1.00; 1.0)]
	6	[(0.72, 0.78, 0.92, 0.97; 0.8), (0.72, 0.78, 0.92, 0.97; 1.0)]	[(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.0)]
		[(0.32, 0.41, 0.58, 0.65; 0.8), (0.32, 0.41, 0.58, 0.65; 1.0)]	[(0.17, 0.22, 0.36, 0.42; 0.8), (0.17, 0.22, 0.36, 0.42; 1.0)]
	7	[(0.04, 0, 10.0.18, 0.23; 0.8), (0.04, 0.10, 0.18, 0.23; 1.0)]	[(0.72, 0.78, 0.92, 0.97; 0.8), (0.72, 0.78, 0.92, 0.97; 1.0)]
		[(0.32, 0.41, 0.58, 0.65; 0.8), (0.32, 0.41, 0.58, 0.65; 1.0)]	[(0.17, 0.22, 0.36, 0.42; 0.8), (0.17, 0.22, 0.36, 0.42; 1.0)]
	8	[(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.0)]	[(0.17, 0.22, 0.36, 0.42; 0.8), (0.17, 0.22, 0.36, 0.42; 1.0)]
		[(0.32, 0.41, 0.58, 0.65; 0.8), (0.32, 0.41, 0.58, 0.65; 1.0)]	[(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.0)]
	9	[(0.72, 0.78, 0.92, 0.97; 0.8), (0.72, 0.78, 0.92, 0.97; 1.0)]	[(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.0)]
		[(0.32, 0.41, 0.58, 0.65; 0.8), (0.32, 0.41, 0.58, 0.65; 1.0)]	[(0.04, 0.10.0.18, 0.23; 0.8), (0.04, 0.10.0.18, 0.23; 1.0)]
	10	[(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.0)]	[(0.32, 0.41, 0.58, 0.65; 0.8), (0.32, 0.41, 0.58, 0.65; 1.0)]
		[(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.0)]	[(0.32, 0.41, 0.58, 0.65; 0.8), (0.32, 0.41, 0.58, 0.65; 1.0)]

步骤 2 7-37 4

$$U_1 = f[(0{:}398; 0{:}457; 0{:}582; 0{:}636; 0{:}8); (0{:}398; 0{:}457; 0{:}582; 0{:}636; 1{:}0)];$$
$$[(0{:}222; 0{:}272; 0{:}388; 0{:}446; 0{:}8); (0{:}222; 0{:}272; 0{:}388; 0{:}446; 1{:}0)];$$
$$[(0{:}280; 0{:}337; 0{:}480; 0{:}542; 0{:}8); (0{:}280; 0{:}337; 0{:}480; 0{:}542; 1{:}0)];$$
$$[(0{:}450; 0{:}481; 0{:}548; 0{:}581; 0{:}8); (0{:}450; 0{:}481; 0{:}548; 0{:}581; 1{:}0)]g;$$

$$U_2 = f[(0{:}488; 0{:}547; 0{:}690; 0{:}746; 0{:}8); (0{:}488; 0{:}547; 0{:}690; 0{:}746; 1{:}0)];$$
$$[(0{:}529; 0{:}590; 0{:}748; 0{:}807; 0{:}8); (0{:}529; 0{:}590; 0{:}748; 0{:}807; 1{:}0)];$$
$$[(0{:}357; 0{:}435; 0{:}602; 0{:}669; 0{:}8); (0{:}357; 0{:}435; 0{:}602; 0{:}669; 1{:}0)];$$
$$[(0{:}418; 0{:}464; 0{:}548; 0{:}588; 0{:}8); (0{:}418; 0{:}464; 0{:}548; 0{:}588; 1{:}0)]g;$$

表 7-14 10个决策专家关于方案A_3和A_4的区间梯形模糊偏好矢量

		1	2
		3	4
A_3	1	[(0.04, 0.10.0.18, 0.23; 0.8), (0.04, 0.10.0.18, 0.23; 1.0)]	[(0.17, 0.22, 0.36, 0.42; 0.8), (0.17, 0.22, 0.36, 0.42; 1.0)]
		[(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.0)]	[(0.93, 0.98, 1.00, 1.00; 0.8), (0.93, 0.98, 1.00, 1.00; 1.0)]
	2	[(0.32, 0.41, 0.58, 0.65; 0.8), (0.32, 0.41, 0.58, 0.65; 1.0)]	[(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.0)]
		[(0.32, 0.41, 0.58, 0.65; 0.8), (0.32, 0.41, 0.58, 0.65; 1.0)]	[(0.17, 0.22, 0.36, 0.42; 0.8), (0.17, 0.22, 0.36, 0.42; 1.0)]
	3	[(0.04, 0.10.0.18, 0.23; 0.8), (0.04, 0.10.0.18, 0.23; 1.0)]	[(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.0)]
		[(0.00, 0.00, 0.02, 0.07; 0.8), (0.00, 0.00, 0.02, 0.07; 1.0)]	[(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.0)]
	4	[(0.32, 0.41, 0.58, 0.65; 0.8), (0.32, 0.41, 0.58, 0.65; 1.0)]	[(0.72, 0.78, 0.92, 0.97; 0.8), (0.72, 0.78, 0.92, 0.97; 1.0)]
		[(0.32, 0.41, 0.58, 0.65; 0.8), (0.32, 0.41, 0.58, 0.65; 1.0)]	[(0.32, 0.41, 0.58, 0.65; 0.8), (0.32, 0.41, 0.58, 0.65; 1.0)]
	5	[(0.32, 0.41, 0.58, 0.65; 0.8), (0.32, 0.41, 0.58, 0.65; 1.0)]	[(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.0)]
		[(0.04, 0.10, 0.18, 0.23; 0.8), (0.04, 0.10, 0.18, 0.23; 1.0)]	[(0.32, 0.41, 0.58, 0.65; 0.8), (0.32, 0.41, 0.58, 0.65; 1.0)]
	6	[(0.72, 0.78, 0.92, 0.97; 0.8), (0.72, 0.78, 0.92, 0.97; 1.0)]	[(0.72, 0.78, 0.92, 0.97; 0.8), (0.72, 0.78, 0.92, 0.97; 1.0)]
		[(0.32, 0.41, 0.58, 0.65; 0.8), (0.32, 0.41, 0.58, 0.65; 1.0)]	[(0.93, 0.98, 1.00, 1.00; 0.8), (0.93, 0.98, 1.00, 1.00; 1.0)]
	7	[(0.00, 0.00, 0.02, 0.07; 0.8), (0.00, 0.00, 0.02, 0.07; 1.0)]	[(0.17, 0.22, 0.36, 0.42; 0.8), (0.17, 0.22, 0.36, 0.42; 1.0)]
		[(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.0)]	[(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.0)]
	8	[(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.0)]	[(0.17, 0.22, 0.36, 0.42; 0.8), (0.17, 0.22, 0.36, 0.42; 1.0)]
		[(0.32, 0.41, 0.58, 0.65; 0.8), (0.32, 0.41, 0.58, 0.65; 1.0)]	[(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.0)]
	9	[(0.17, 0.22, 0.36, 0.42; 0.8), (0.17, 0.22, 0.36, 0.42; 1.0)]	[(0.00, 0.00, 0.02, 0.07; 0.8), (0.00, 0.00, 0.02, 0.07; 1.0)]
		[(0.17, 0.22, 0.36, 0.42; 0.8), (0.17, 0.22, 0.36, 0.42; 1.0)]	[(0.00, 0.00, 0.00, 0.00; 0.8), (0.00, 0.00, 0.00, 0.00; 1.0)]
	10	[(0.72, 0.78, 0.92, 0.97; 0.8), (0.72, 0.78, 0.92, 0.97; 1.0)]	[(0.04, 0.10, 0.18, 0.23; 0.8), (0.04, 0.10, 0.18, 0.23; 1.0)]
		[(0.17, 0.22, 0.36, 0.42; 0.8), (0.17, 0.22, 0.36, 0.42; 1.0)]	[(0.17, 0.22, 0.36, 0.42; 0.8), (0.17, 0.22, 0.36, 0.42; 1.0)]
A_4	1	[(0.17, 0.22, 0.36, 0.42; 0.8), (0.17, 0.22, 0.36, 0.42; 1.0)]	[(0.04, 0.10, 0.18, 0.23; 0.8), (0.04, 0.10, 0.18, 0.23; 1.0)]
		[(0.72, 0.78, 0.92, 0.97; 0.8), (0.72, 0.78, 0.92, 0.97; 1.0)]	[(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.0)]
	2	[(0.04, 0.10, 0.18, 0.23; 0.8), (0.04, 0.10, 0.18, 0.23; 1.0)]	[(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.0)]
		[(0.17, 0.22, 0.36, 0.42; 0.8), (0.17, 0.22, 0.36, 0.42; 1.0)]	[(0.00, 0.00, 0.00, 0.00; 0.8), (0.00, 0.00, 0.00, 0.00; 1.0)]
	3	[(0.93, 0.98, 1.00, 1.00; 0.8), (0.93, 0.98, 1.00, 1.00; 1.0)]	[(0.32, 0.41, 0.58, 0.65; 0.8), (0.32, 0.41, 0.58, 0.65; 1.0)]
		[(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.0)]	[(1.00, 1.00, 1.00, 1.00; 0.8), (1.00, 1.00, 1.00, 1.00; 1.0)]
	4	[(0.04, 0.10, 0.18, 0.23; 0.8), (0.04, 0.10, 0.18, 0.23; 1.0)]	[(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.0)]
		[(0.32, 0.41, 0.58, 0.65; 0.8), (0.32, 0.41, 0.58, 0.65; 1.0)]	[(0.00, 0.00, 0.02, 0.07; 0.8), (0.00, 0.00, 0.02, 0.07; 1.0)]
	5	[(0.32, 0.41, 0.58, 0.65; 0.8), (0.32, 0.41, 0.58, 0.65; 1.0)]	[(0.32, 0.41, 0.58, 0.65; 0.8), (0.32, 0.41, 0.58, 0.65; 1.0)]
		[(0.17, 0.22, 0.36, 0.42; 0.8), (0.17, 0.22, 0.36, 0.42; 1.0)]	[(0.00, 0.00, 0.02, 0.07; 0.8), (0.00, 0.00, 0.02, 0.07; 1.0)]
	6	[(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.0)]	[(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.0)]
		[(0.32, 0.41, 0.58, 0.65; 0.8), (0.32, 0.41, 0.58, 0.65; 1.0)]	[(0.17, 0.22, 0.36, 0.42; 0.8), (0.17, 0.22, 0.36, 0.42; 1.0)]
	7	[(0.32, 0.41, 0.58, 0.65; 0.8), (0.32, 0.41, 0.58, 0.65; 1.0)]	[(0.17, 0.22, 0.36, 0.42; 0.8), (0.17, 0.22, 0.36, 0.42; 1.0)]
		[(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.0)]	[(0.32, 0.41, 0.58, 0.65; 0.8), (0.32, 0.41, 0.58, 0.65; 1.0)]
	8	[(0.17, 0.22, 0.36, 0.42; 0.8), (0.17, 0.22, 0.36, 0.42; 1.0)]	[(0.17, 0.22, 0.36, 0.42; 0.8), (0.17, 0.22, 0.36, 0.42; 1.0)]
		[(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.0)]	[(0.72, 0.78, 0.92, 0.97; 0.8), (0.72, 0.78, 0.92, 0.97; 1.0)]
	9	[(0.04, 0.10, 0.18, 0.23; 0.8), (0.04, 0.10, 0.18, 0.23; 1.0)]	[(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.0)]
		[(0.17, 0.22, 0.36, 0.42; 0.8), (0.17, 0.22, 0.36, 0.42; 1.0)]	[(0.17, 0.22, 0.36, 0.42; 0.8), (0.17, 0.22, 0.36, 0.42; 1.0)]
	10	[(0.72, 0.78, 0.92, 0.97; 0.8), (0.72, 0.78, 0.92, 0.97; 1.0)]	[(0.72, 0.78, 0.92, 0.97; 0.8), (0.72, 0.78, 0.92, 0.97; 1.0)]
		[(0.17, 0.22, 0.36, 0.42; 0.8), (0.17, 0.22, 0.36, 0.42; 1.0)]	[(0.04, 0.10, 0.18, 0.23; 0.8), (0.04, 0.10, 0.18, 0.23; 1.0)]

U_3 = f[(0:323; 0:384; 0:512; 0:570; 0:8); (0:323; 0:384; 0:512; 0:570; 1:0)];
[(0:388; 0:440; 0:574; 0:631; 0:8); (0:388; 0:440; 0:574; 0:631; 1:0)];
[(0:297; 0:363; 0:506; 0:569; 0:8); (0:297; 0:363; 0:506; 0:569; 1:0)];
[(0:458; 0:511; 0:630; 0:679; 0:8); (0:458; 0:511; 0:630; 0:679; 1:0)]g;

U_4 = f[(0:333; 0:395; 0:514; 0:566; 0:8); (0:333; 0:395; 0:514; 0:566; 1:0)];
[(0:406; 0:466; 0:618; 0:678; 0:8); (0:406; 0:466; 0:618; 0:678; 1:0)];
[(0:378; 0:437; 0:592; 0:653; 0:8); (0:378; 0:437; 0:592; 0:653; 1:0)];
[(0:300; 0:336; 0:424; 0:469; 0:8); (0:300; 0:336; 0:424; 0:469; 1:0)]g:

步骤 3

$$\begin{aligned} V_P = \{&[(0.93, 0.98, 1.00, 1.00; 0.8), (0.93, 0.98, 1.00, 1.00; 1.0)], \\ &[(1.00, 1.00, 1.00, 1.00; 0.8), (1.00, 1.00, 1.00, 1.00; 1.0)], \\ &[(1.00, 1.00, 1.00, 1.00; 0.8), (1.00, 1.00, 1.00, 1.00; 1.0)], \\ &[(0.93, 0.98, 1.00, 1.00; 0.8), (0.93, 0.98, 1.00, 1.00; 1.0)]\}. \end{aligned}$$

步骤 4 7-35

$$T = \begin{pmatrix} 0.1733 & 0.2024 & 0.0625 & 0.1675 \\ 0.3524 & 0.4033 & 0.1284 & 0.1486 \\ 0.0455 & 0.1148 & 0.0192 & 0.2656 \\ 0.0541 & 0.1756 & 0.1270 & 0.0715 \end{pmatrix}.$$

步骤 5

$$\begin{aligned} w_1 &= < C_1, [(0.32, 0.41, 0.58, 0.65; 0.8), (0.32, 0.41, 0.58, 0.65; 1.0)] >, \\ w_2 &= < C_2, [(0.72, 0.78, 0.92, 0.97; 0.8), (0.72, 0.78, 0.92, 0.97; 1.0)] >, \\ w_3 &= < C_3, [(0.17, 0.22, 0.36, 0.42; 0.8), (0.17, 0.22, 0.36, 0.42; 1.0)] >, \\ w_4 &= < C_4, [(0.93, 0.98, 1.00, 1.00; 0.8), (0.93, 0.98, 1.00, 1.00; 1.0)] > . \end{aligned}$$

7-33 $d_1 = 0.5942$ $d_2 = 0.9301$ $d_3 = 0.4830$ $d_4 = 1.0331$ 7-38

$\hat{W} = (0.1954, 0.3059, 0.1589, 0.3398).$

步骤 6 $A = \{A_1, A_2, \ldots, A_4\}$ 7-39

$$O = \hat{W}T = (0.1954, 0.3059, 0.1589, 0.3398) \begin{pmatrix} 0.1733 & 0.2024 & 0.0625 & 0.1675 \\ 0.3524 & 0.4033 & 0.1284 & 0.1486 \\ 0.0455 & 0.1148 & 0.0192 & 0.2656 \\ 0.0541 & 0.1756 & 0.1270 & 0.0715 \end{pmatrix}$$

$$= (0.1673, 0.2408, 0.0977, 0.1447).$$

$$A_2 \succ A_1 \succ A_4 \succ A_3$$

A_2

7.4.4 小结

7.5 本章小结

8 结论与展望

8.1 本文结论

8.1.1 主要工作

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J- E- C-

J- E-

C- E-

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8.1.2 创新点

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$$
\begin{aligned}
S(X;Y) &= \alpha \frac{2XY}{\|X\|_2^2 + \|Y\|_2^2} + (1-\alpha)\frac{XY}{\|X\|_2\|Y\|_2} \\
&= \alpha \frac{2\sum\limits_{i=1} x_i y_i}{\sum\limits_{i=1} x_i^2 + \sum\limits_{i=1} y_i^2} + (1-\alpha)\frac{\sum\limits_{i=1} x_i y_i}{\sqrt{\sum\limits_{i=1} x_i^2}\sqrt{\sum\limits_{i=1} y_i^2}};
\end{aligned}
$$

2

3

$$S_{ij}(X_i;X_j) = \frac{1}{\sqrt{n}}\frac{\|T_i + T_j\|_2}{\|T_i\|_\infty + \|T_j\|_\infty};$$

$$\|T\|_2 = (\rho(T'T))^{\frac{1}{2}}; \quad \|T_i\|_\infty = \max_{1\leq j_1\leq n}\sum_{j=1} |t_{ij}|.$$

4

$$S_V^{FS}(A;B) = \alpha\frac{2AB}{\|A\|_2^2 + \|B\|_2^2} + (1-\alpha)\frac{AB}{\|A\|_2\|B\|_2};$$

$AB = \sum_{i=1} \mu_A(x_i)\mu_B(x_i)$, $\|A\|_2 = \sqrt{\sum_{i=1} \mu_A^2(x_i)}$, $\|B\|_2 = \sqrt{\sum_{i=1} \mu_B^2(x_i)}$, $0 \leq \alpha \leq 1$.

$$S_V^{IFS}(A;B) = \alpha\frac{1}{n}\sum_{i=1}\frac{2(\mu_A(x_i)\mu_B(x_i) + \nu_A(x_i)\nu_B(x_i))}{(\mu_A^2(x_i) + \nu_A^2(x_i)) + (\mu_B^2(x_i) + \nu_B^2(x_i))}$$
$$+(1-\alpha)\frac{1}{n}\sum_{i=1}\frac{\mu_A(x_i)\mu_B(x_i) + \nu_A(x_i)\nu_B(x_i)}{\sqrt{\mu_A^2(x_i) + \nu_A^2(x_i)}\sqrt{\mu_B^2(x_i) + \nu_B^2(x_i)}}.$$

$$S^J(\alpha;\beta) = \frac{\sum_{i=1} a_i b_i}{\sum_{i=1} a_i^2 + \sum_{i=1} b_i^2 - \sum_{i=1} a_i b_i};$$

$$S^E(\alpha;\beta) = \frac{2\sum_{i=1} a_i b_i}{\sum_{i=1} a_i^2 + \sum_{i=1} b_i^2};$$

$$S^C(\alpha;\beta) = \frac{\sum_{i=1} a_i b_i}{\sqrt{\sum_{i=1} a_i^2}\sqrt{\sum_{i=1} b_i^2}}.$$

$$S^H(\alpha_1,\alpha_2) = 1 - \frac{\sum_{i=1} |a_{1i} - a_{2i}|}{3} - \frac{\sum_{i=1} |b_{1i} - b_{2i}|}{3},$$
$$S^E(\alpha_1,\alpha_2) = 1 - \sqrt{\frac{\sum_{i=1} (a_{1i} - a_{2i})^2}{3} + \frac{\sum_{i=1} (b_{1i} - b_{2i})^2}{3}},$$

$$S^F(\alpha_1,\alpha_2) = 1 - max\left\{\frac{\sum_{i=1} |a_{1i} - a_{2i}|}{3}, \frac{\sum_{i=1} |b_{1i} - b_{2i}|}{3}\right\}.$$

$$S_O = \sqrt{\sum_i (a_i - b_i)^2}.$$

$$S^{12} = S(E^1,E^2) = \frac{1}{2}\frac{2E^1E^2}{\|E^1\|_2 + \|E^2\|_2} + \frac{1}{2}\frac{2E^1E^2}{\|E^1\|_2\|E^2\|_2}$$
$$= \frac{1}{2}\frac{\sum_{i=1} e_i^1 e_i^2}{\sum_{i=1} (e_i^1)^2 + \sum_{i=1} (e_i^2)^2} + \frac{1}{2}\frac{\sum_{i=1} e_i^1 e_i^2}{\sqrt{\sum_{i=1} (e_i^1)^2}\sqrt{\sum_{i=1} (e_i^2)^2}}$$

$$S(\alpha,\beta) = 1 - min(\omega_{\alpha^L}, \omega_{\beta^L}) \times \frac{\sum_{i=1} |a_i^L - b_i^L|}{4}$$
$$- min(\omega_{\alpha^U}, \omega_{\beta^U}) \times \frac{\sum_{i=1} |a_i^U - b_i^U|}{4}.$$

5

TOPSIS

8.1.3 研究不足

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```
Yi=Y(i,:);
Z(i,j)=Y(i,j)/norm(Yi);
end
end Z
for i=1:8,j=1:12
yij=Y(i,j,:,:);
yi=Y(i,:),i=1:8;
Z(i,j)=Y(i,j)/sqrt(Yj);
end
| -Z(:,:,i,j)=Zij | {
| | 4.5 | |
| | -2010.6.23 | -
| - V | {
clc
clear V=[ 0.8704 0.9043 1.0000 1.0000 0.8398 0.9346 0.5320 0.6662 0.5492
0.6737 0.6737 0.6182 0.8838 0.7246 0.7924 0.4522 0.7688 0.9743 1.0000 0.0065
0.4149 0.2742 0.6737 0.0845 0.7329 0.9378 0.6713 0.8307 0.7618 1.0000 0.2559
1.0000 0.1458 0.3049 0.4463 0.5910 0.6824 0.7783 0.5482 0.5831 0.4522 0.4445
0.9209 0.9261 0.6729 0.8721 1.0000 0.7912 1.0000 0.8591 0.7748 0.9018 0.6326
0.8777 0.9820 0.7783 0.8299 0.7540 0.6019 0.4788 0.4434 0.2446 0.3255 0.3811
0.3808 0.3035 0.2694 0.6189 0.4979 0.4542 0.2857 0.4632 0.1628 0.3042 0.4236
0.4998 0.3327 0.2700 0.5309 0.4764 0.4226 0.3975 0.5468 0.6502 0.6263 0.6316
0.6713 0.7263 0.8021 0.8153 0.4422 0.0873 0.4979 0.2435 0.2857 0.6502 0.8586
0.8724 0.5482 0.6875 0.6737 0.6517 0.4849 0.8724 0.4290 0.3975 0.5468 0.6502
0.4260 0.5859 0.4012 0.5934 0.6120 0.5294 0.5000 0.4759 0.6459 0.7130 0.3808
0.2972 0.3059 0.5242 0.5995 0.5576 0.7393 0.4819 0.4798 0.3632 0.6431 0.7356
0.3808 0.4741 0.2178 0.1016 0.4979 0.2108 0.1976 0.1695 0.3423 0.9612 0.4979
0.7852 0.7812 0.8317 0.4585 0.2446 0.5771 0.1351 0.4359 0.0432 0.5466 0.4355
0.6975 0.7852 0.8467 0.9579 0.4024 0.2446 0.6024 0.5836 0.3808 0.2146 0.4400
0.4355 0.5492 0.6660 0.6737 0.7211 0.6600 0.6975 0.6751 0.1458 0.5856 0.4803
0.6420 0.3414 0.4979 0.7550 0.3808 0.6502 ]; | -V(i,:)=Vi; |
| | 30 Ai | | {
R=0.5; | -
for i=1:30
vi=V(i,:);
for j=1:5
for k=1:5
```

```
if log(vi(j)+vi(k))>=R  |              R | -
 | -A                          i              Vi                    Ai | -
A(j,k,i)=1;
else
A(j,k,i)=0;
end
end
end
end  | -A(:,:,i)=Ai | {
 |  | -   30                  Ai              |  |  | {
k=0;  |            | -
r=input('                        r=') ; | -              | -
a=zeros(30,30); |                                  |
N=linspace(1,30,30); |              T    30                  |
t=0;
while (sum(N)>0&&k<30)  |                         0                 |
for i=1:length(N)
if i==1
C=A(:,:,N(i));  |                  C | -
k=k+1;  |                  1 |
a(k,N(i))=N(i);
N(i)=0;                        ,                        |
else if norm(A(:,:,N(i))+C,2)/(norm(A(:,:,N(i)),inf)+norm(C,inf))>=r;
a(k,N(i))=N(i);
C=max(C,A(:,:,N(i)));  |                 C                           |
N(i)=0;                        ,                        |
else
continue end
end
L=N>0;
is logical(L);
N=N(L);
end
 |  |  |               |  |
for i=1:30
```

```
C(6)=(A(6,:)*B(6,:)')/(norm(A(6;:)) ¤ norm(B(6;:))^2)
V(6)=b*E(6)+(1-b)*C(6)
 | {5.4                  |  |
 |  | -2012.3.19 | -
 | -                  A | {
clc
clear
A=[ 0.6 0.4 0.55 0.35 0.4 0.2 0.52 0.45 ];
B=[ 0.55 0.35 0.65 0.3 0.55 0.04 0.45 0.15 ];
M=[ 0.6 0.3 0.68 0.3 0.72 0.25 0.65 0.12];
P=[ 0.6 0.3 0.68 0.3 0.72 0.04 0.65 0.12];
Q=[0.55 0.40 0.55 0.3 0.4 0.25 0.45 0.45];
E1=2*(A(1,:)*P(1,:)')/(norm(A(1;:))^2 + norm(P(1;:))^2);
E2=2*(A(2,:)*P(2,:)')/(norm(A(2;:))^2 + norm(P(2;:))^2);
E3=2*(A(3,:)*P(3,:)')/(norm(A(3;:))^2 + norm(P(3;:))^2);
E4=2*(A(4,:)*P(4,:)')/(norm(A(4;:))^2 + norm(P(4;:))^2);
PE(1)=1/4*(E1+E2+E3+E4);
E1=2*(B(1,:)*P(1,:)')/(norm(B(1;:))^2 + norm(P(1;:))^2);
E2=2*(B(2,:)*P(2,:)')/(norm(B(2;:))^2 + norm(P(2;:))^2);
E3=2*(B(3,:)*P(3,:)')/(norm(B(3;:))^2 + norm(P(3;:))^2);
E4=2*(B(4,:)*P(4,:)')/(norm(B(4;:))^2 + norm(P(4;:))^2);
PE(2)=1/4*(E1+E2+E3+E4);
E1=2*(M(1,:)*P(1,:)')/(norm(M(1;:))^2 + norm(P(1;:))^2);
E2=2*(M(2,:)*P(2,:)')/(norm(M(2;:))^2 + norm(P(2;:))^2);
E3=2*(M(3,:)*P(3,:)')/(norm(M(3;:))^2 + norm(P(3;:))^2);
E4=2*(M(4,:)*P(4,:)')/(norm(M(4;:))^2 + norm(P(4;:))^2);
PE(3)=1/4*(E1+E2+E3+E4)
E1=2*(A(1,:)*Q(1,:)')/(norm(A(1;:))^2 + norm(Q(1;:))^2);
E2=2*(A(2,:)*Q(2,:)')/(norm(A(2;:))^2 + norm(Q(2;:))^2);
E3=2*(A(3,:)*Q(3,:)')/(norm(A(3;:))^2 + norm(Q(3;:))^2);
E4=2*(A(4,:)*Q(4,:)')/(norm(A(4;:))^2 + norm(Q(4;:))^2);
NE(1)=1/4*(E1+E2+E3+E4);
E1=2*(B(1,:)*Q(1,:)')/(norm(B(1;:))^2 + norm(Q(1;:))^2);
E2=2*(B(2,:)*Q(2,:)')/(norm(B(2;:))^2 + norm(Q(2;:))^2);
E3=2*(B(3,:)*Q(3,:)')/(norm(B(3;:))^2 + norm(Q(3;:))^2);
```

```
E4=2*(B(4,:)*Q(4,:)')/(norm(B(4;:))^2 + norm(Q(4;:))^2);
NE(2)=1/4*(E1+E2+E3+E4);
E1=2*(M(1,:)*Q(1,:)')/(norm(M(1;:))^2 + norm(Q(1;:))^2);
E2=2*(M(2,:)*Q(2,:)')/(norm(M(2;:))^2 + norm(Q(2;:))^2);
E3=2*(M(3,:)*Q(3,:)')/(norm(M(3;:))^2 + norm(Q(3;:))^2);
E4=2*(M(4,:)*Q(4,:)')/(norm(M(4;:))^2 + norm(Q(4;:))^2);
NE(3)=1/4*(E1+E2+E3+E4)
de(1)=PE(1)/NE(1);
de(2)=PE(2)/NE(2);
de(3)=PE(3)/NE(3)
 | | 5.5 | |
 | | -2012.3.25 | -
 | - A | {
clc
clear
A= [0.0539 0.0911 0.2953 0.3302 0.0855 0.4693 0.1357 0.0504 0.2786 0.3862
0.0716 0.2797 0.4161 0.3087 0.0611 0.3356 0.3145 0.0508 0.4171 0.0078 0.3921
0.3528 0.1800 0.2271 0.2428 0.4472 0.2585 0.2783 0.1010 0.2269 0.4984 0.2768
0.1716 0.4680 0.3308 0.2585 0.3469 0.4726 0.4181 0.3657 0.4176 0.1612 0.3601
0.1734 0.4097 0.2659 0.4431 0.4656 0.3783 0.2070 0.0355 0.4439 0.4871 0.2854
0.2000 0.4159 0.8147 0.1270 0.6324 0.2785 0.7690 0.0357 0.7060 0.2769 0.0971
0.6948 0.3922 0.1712 0.0344 0.3816 0.1869 0.4456 0.7094 0.2760 0.6551 0.1190
0.6797 0.1626 0.4984 0.3404 0.2238 0.2551 0.5472 0.1493 0.6160 0.1386 0.2575
0.2543 0.2435 0.3500 0.2511 0.4733 0.3804 0.5678 0.0119 0.3371 0.1622 0.7943
0.3112 0.5285 0.1656 0.6020 0.2630 0.6541 0.4505 0.0838 0.1524 0.8258 0.0782
0.4427 0.0067 0.9619 0.0046 0.7749 0.0844 0.3998 0.1818 0.2638 0.1455 0.1361
0.5499 0.1450 0.3510 0.5132 0.4018 0.0760 0.2399 0.1233 0.1839 0.2400 0.4173
0.0497 0.4909 0.4893 0.3692 0.1112 0.2417 0.4039 0.0965 0.1320 0.5752 0.0598
0.2348 0.3532 0.8212 0.0154 0.0430 0.7317 0.6477 0.3509 0.5470 0.2963 0.7447
0.1890 0.6868 0.1835 0.3685 0.6256 0.7802 0.0811 0.9294 0.0057 0.4868 0.4359
0.4468 0.3063 0.5085 0.4108 0.8176 0.0948 0.6443 0.3186 0.3507 0.6390 0.2077
0.3012 0.4709 0.2305 0.7443 0.1948 0.2259 0.1707 0.2277 0.4357 0.0311 0.9234
0.4302 0.1848 0.4389 0.1111 0.2581 0.4087 0.5949 0.2622 0.2217 0.1174 0.2967
0.3188 0.4242 0.5079 0.9294 0.0057 0.7443 0.0078 0.7690 0.0598 0.7802 0.0504
0.0046 0.9234 0.0430 0.8358 0.0782 0.6390 0.0119 0.9616 ];
for i=1:27
b(i)=A(i,1)-A(i,2);
```

```
c(i)=A(i,3)-A(i,4);
d(i)=A(i,5)-A(i,6);
e(i)=A(i,7)-A(i,8);
end
Y=[b',c',d',e']';
Y1=min(Y,[],1);
Y2=max(Y,[],1);
for i=1:4
for j=1:27
Z(i,j)=[Y(i,j)-Y1(j)]/[Y2(j)-Y1(j)];
end
end
z=Z';
w=[0.1 0.3 0.2 0.4 0.1];
A1=[z(1,:);z(2,:);z(3,:);z(4,:);z(5,:)];
A2=[z(6,:);z(7,:);z(8,:);z(9,:);z(10,:)];
A3=[z(11,:);z(12,:);z(13,:);z(14,:);z(15,:)];
A4=[z(16,:);z(17,:);z(18,:);z(19,:);z(20,:)];
A5=[z(21,:);z(22,:);z(23,:);z(24,:);z(25,:)];
s1=w*A1; s2=w*A2; s3=w*A3; s4=w*A4; s5=w*A5;
 | | 5.5          | |
 | | -2012.3.20 | -
 | -          A | {
clc
clear
A =[0.7389 0.7438 0 1.0000 0.4370 0.1739 1.0000 0 0.6810 1.0000 0.1893 0
0 0.4333 0.1843 1.0000 0 1.0000 0.4630 0.9460 1.0000 0.7501 0.5963 0 0.9502 0
1.0000 0.3155 0.8798 0 1.0000 0.1982 0 1.0000 0.3060 0.4141 0.7284 1.0000 0.9497
0 0 0.8437 1.0000 0.0678 1.0000 0.4710 0.6301 0 0 1.0000 0.4718 0.5810 1.0000
0.2132 0.4469 0 0 0.5834 0.8828 1.0000 1.0000 0 0.8605 0.4916 0 1.0000 0.1362
0.7413 0 0.1870 1.0000 0.0646 1.0000 0 0.6594 0.6286 0.8500 0.7952 0 1.0000
1.0000 0 0.1027 0.0536 1.0000 0.6073 0 0.1927 0.5919 1.0000 0.3475 0 0 0.9325
1.0000 0.6079 1.0000 0.4515 0.1479 0 ];
P=[1.0000 0.1273 0 0.0960];
N=[ 0.0795 0.4034 1.0000 0];
for i=1:25
```

```
SP(i)=A(i,:)*P'/(norm(A(i,:))+norm(P));
SN(i)=A(i,:)*N'/(norm(A(i,:))+norm(N));
R(i)=1/2*(SP(i)+SN(i));
end
x=linspace(0,0.1,25); plot(x,SP,'rp-',x,SN,'b*-',x,R,'g>-');
legend('Positive-similarity','Negitive-similarity','Reletive-similarity');
```

| | 6.2 | |

| | -2012.5.12 | -

| -   V | {

```
clc
clear
A= [0.1300 0.1700 0.2200 0.3600 0.4600 0.5600 0.3900 0.5400 0.6900 0.4200 0.5000 0.6000 0.5200 0.6000 0.6000 ];
B=[0.2200 0.2700 0.3200 0.5400 0.6400 0.7400 0.6000 0.7500 0.9000 0.4400 0.5400 0.6200 0.1800 0.2600 0.2600];
C=[0.1800 0.2300 0.2700 0.2800 0.3600 0.4600 0.6900 0.8400 0.9900 0.1600 0.2400 0.3400 0.4800 0.5800 0.5800];
D=[0.2100 0.2600 0.3100 0.3400 0.4200 0.5200 0.5700 0.7200 0.8700 0.2800 0.3800 0.4800 0.4600 0.5600 0.5600];
P=[0.40 0.50 0.60 0.70 0.80 0.90 0.80 0.90 1.00 0.70 0.80 0.90 0.50 0.60 0.70];
for i=1:5
J1(i)=A(i,:)*P(i,:)'/(norm(A(i;:))^2 + norm(P(i;:))^2-A(i,:)*P(i,:)');
end
for i=1:5
J2(i)=B(i,:)*P(i,:)'/(norm(B(i;:))^2 + norm(P(i;:))^2-B(i,:)*P(i,:)');
end for i=1:5
J3(i)=C(i,:)*P(i,:)'/(norm(C(i;:))^2 + norm(P(i;:))^2-C(i,:)*P(i,:)');
end
for i=1:5
J4(i)=D(i,:)*P(i,:)'/(norm(D(i;:))^2 + norm(P(i;:))^2-D(i,:)*P(i,:)');
end
for i=1:5
E1(i)=2*A(i,:)*P(i,:)'/(norm(A(i;:))^2 + norm(P(i;:))^2);
end
for i=1:5
```

```
E2(i)=2*B(i,:)*P(i,:)'/(norm(B(i;:))^2 + norm(P(i;:))^2);
end
for i=1:5
E3(i)=2*C(i,:)*P(i,:)'/(norm(C(i;:))^2 + norm(P(i;:))^2);
end
for i=1:5
E4(i)=2*D(i,:)*P(i,:)'/(norm(D(i;:))^2 + norm(P(i;:))^2);
end
for i=1:5
C1(i)=A(i,:)*P(i,:)'/(norm(A(i,:))*norm(P(i,:)));
end
for i=1:5
C2(i)=B(i,:)*P(i,:)'/(norm(B(i,:))*norm(P(i,:)));
end
for i=1:5 C3(i)=C(i,:)*P(i,:)'/(norm(C(i,:))*norm(P(i,:)));
end
for i=1:5
C4(i)=D(i,:)*P(i,:)'/(norm(D(i,:))*norm(P(i,:)));
end
J=[J1; J2;J3;J4]';
E=[E1;E2;E3;E4]';
C=[C1;C2;C3;C4]';
W=[0.1481,0.2222,0.0741,0.3580,0.1975];
j=W*J; e=W*E; c=W*C.
```

| | 6.3 | |

| | -2012.7.30 | -

| - B | {

```
clc
clear
B =[0.2400 0.4400 0.3400 0.4200 0.3200 0.5400 0.4200 0.5200 0.4200 0.6400
0.5200 0.6200 0.1600 0.3600 0.2600 0.3400 0.3600 0.6000 0.4400 0.5600 0.5200
0.7400 0.6000 0.7200 0.3600 0.5200 0.2800 0.3400 0.4600 0.6200 0.3600 0.4200
0.5600 0.7200 0.4400 0.5200 0.2800 0.4600 0.2000 0.2600 0.4800 0.6800 0.3800
0.4600 0.6600 0.8000 0.5400 0.6200 0.2400 0.3600 0.4200 0.3800 0.3200 0.4600
0.5200 0.4800 0.4200 0.5600 0.6200 0.5800 0.1600 0.2600 0.3400 0.2800 0.3600
```

0.5000 0.5800 0.5200 0.5200 0.6600 0.7200 0.6800 0.4000 0.4400 0.1600 0.2200 0.4800 0.5000 0.2000 0.3000 0.5800 0.6000 0.2800 0.4000 0.3400 0.4000 0.1200 0.1400 0.5600 0.5600 0.2400 0.3400 0.6600 0.6600 0.3800 0.5000 0.5200 0.1800 0.4600 0.4400 0.5800 0.2200 0.5200 0.5400 0.6800 0.3000 0.6200 0.6400 0.4800 0.1600 0.4200 0.3800 0.6800 0.2800 0.6000 0.5800 0.7600 0.3800 0.7000 0.7200];

A=[0.0 0.0 0.1 0.0 0.1 0.2 0.1 0.2 0.3 0.0 0.2 0.4 0.2 0.3 0.4 0.1 0.3 0.5 0.1 0.2 0.3 0.0 0.2 0.4 0.0 0.0 0.1 0.0 0.1 0.2];

C=A';

a1=abs(B(:,1)-C)'; a2=abs(B(:,2)-C)';

a3=abs(B(:,3)-C)'; a4=abs(B(:,4)-C)';

b1=[0.2400 0.3200 0.3200 0.1600 0.2600 0.3200 0.2600 0.2600 0.2600 0.2800 0.2800 0.2600 0.0400 0.0200 0.0200 0.0600 0.0600 0.0200 0.3000 0.2800 0.2800 0.3400 0.3600 0.2600 0.5200 0.5800 0.5800 0.4800 0.5800 0.5600];

b2=[0.4400 0.5400 0.5400 0.3600 0.5000 0.5400 0.4200 0.4200 0.4200 0.4600 0.4800 0.4000 0.1600 0.1600 0.1600 0.1600 0.2000 0.1600 0.3400 0.3000 0.3000 0.4000 0.3600 0.2600 0.1800 0.2200 0.2000 0.1600 0.1800 0.1800];

b3=[0.3400 0.4200 0.4200 0.2600 0.3400 0.4000 0.1800 0.1600 0.1400 0.2000 0.1800 0.1400 0.2200 0.2200 0.2200 0.2400 0.2800 0.2200 0.0600 0 0.0200 0.1200 0.0400 0.0200 0.4600 0.5200 0.5200 0.4200 0.5000 0.5000];

b4=[0.4200 0.5200 0.5200 0.3400 0.4600 0.5200 0.2400 0.2200 0.2200 0.2600 0.2600 0.2200 0.1800 0.1800 0.1800 0.1800 0.2200 0.1800 0.1200 0.1000 0.1000 0.1400 0.1400 0.1000 0.4400 0.5400 0.5400 0.3800 0.4800 0.5200];

for i=1:5

for j=1:6

E(i,j)=($b1(i;j)^2$);

F(i,j)=($b2(i;j)^2$);

G(i,j)=($b3(i;j)^2$);

H(i,j)=($b4(i;j)^2$);

end

end

E =[0.0576 0.1024 0.1024 0.0256 0.0676 0.1024 0.0676 0.0676 0.0676 0.0784 0.0784 0.0676 0.0016 0.0004 0.0004 0.0036 0.0036 0.0004 0.0900 0.0784 0.0784 0.1156 0.1296 0.0676 0.2704 0.3364 0.3364 0.2304 0.3364 0.3136];

F =[0.1936 0.2916 0.2916 0.1296 0.2500 0.2916 0.1764 0.1764 0.1764 0.2116 0.2304 0.1600 0.0256 0.0256 0.0256 0.0256 0.0400 0.0256 0.1156 0.0900 0.0900 0.1600 0.1296 0.0676 0.0324 0.0484 0.0400 0.0256 0.0324 0.0324];

```
G =[0.1156 0.1764 0.1764 0.0676 0.1156 0.1600 0.0324 0.0256 0.0196 0.0400 0.0324 0.0196 0.0484 0.0484 0.0484 0.0576 0.0784 0.0484 0.0036 0 0.0004 0.0144 0.0016 0.0004 0.2116 0.2704 0.2704 0.1764 0.2500 0.2500];
H =[ 0.1764 0.2704 0.2704 0.1156 0.2116 0.2704 0.0576 0.0484 0.0484 0.0676 0.0676 0.0484 0.0324 0.0324 0.0324 0.0324 0.0484 0.0324 0.0144 0.0100 0.0100 0.0196 0.0196 0.0100 0.1936 0.2916 0.2916 0.1444 0.2304 0.2704];
e1=(E(:, 1)+E(:,2)+E(:,3)+E(:,4)+E(:, 5)+E(:,6)) ;
e2=(F(:, 1)+F(:,2)+F(:,3)+F(:,4)+F(:, 5)+F(:,6));
e3=(G(:, 1)+G(:,2)+G(:,3)+G(:,4)+G(:, 5)+G(:,6)) ;
e4=(H(:, 1)+H(:,2)+H(:,3)+H(:,4)+H(:, 5)+H(:,6)) ;
f1=sqrt(e1(:,1)); f2=sqrt(e2(:,1));
f3=sqrt(e3(:,1)); f4=sqrt(e4(:,1));
a=[f1,f2,f3,f4];
w=[0.2315 0.0741 0.2315 0.3519 0.1111];
O=w*a;
 | | 7.1            | |
 | | -2012.4.15 | -
 | -         D | {
clc
clear
D =[0.0881 0.0991 0.1321 0.1651 0.1321 0.1651 0.1982 0.2312 0.2037 0.2312 0.2642 0.2973 0.1321 0.1651 0.1982 0.2312 0.1522 0.1776 0.2030 0.2283 0.1395 0.1522 0.1776 0.1987 0.1268 0.1480 0.1734 0.1987 0.1268 0.1438 0.1691 0.1945 0.0656 0.0805 0.0983 0.1162 0.0954 0.1073 0.1252 0.1430 0.1043 0.1132 0.1311 0.1460 0.0298 0.0447 0.0626 0.0805 0.0672 0.0830 0.1067 0.1305 0.0870 0.1028 0.1265 0.1463 0.0237 0.0395 0.0633 0.0870 0.1305 0.1502 0.1739 0.1937];
a1=max(D(1,:),D(2,:)); a2=max(D(3,:),D(4,:));` a=max(a1,a2);
b1=min(D(1,:),D(2,:)); b2=min(D(3,:),D(4,:)); b=min(b1,b2);
c1=abs(D(1,:)-a); c2=abs(D(2,:)-a);
c3=abs(D(3,:)-a); c4=abs(D(4,:)-a);
C=[c1;c2;c3;c4];
for i=1:4
for j=1:16
E(i,j)=(C(i,j)^2);
end
end
```

```
e1=(E(:, 1)+E(:,2)+E(:,3)+E(:,4));
e2=(E(:, 5)+E(:,6)+E(:,7)+E(:,8));
e3=(E(:, 9)+E(:,10)+E(:,11)+E(:,12));
e4=(E(:, 13)+E(:,14)+E(:,15)+E(:,16));
f1=sqrt(e1(:,1));  f2=sqrt(e2(:,1));
f3=sqrt(e3(:,1));  f4=sqrt(e4(:,1));
F=[f1, f2, f3, f4];
g=sum(F') G=1/4*sum(F');
h1=abs(D(1,:)-b); h2=abs(D(2,:)-b);
h3=abs(D(3,:)-b); h4=abs(D(4,:)-b);
H=[h1;h2;h3;h4];
for i=1:4
for j=1:16
K(i,j)=(H(i;j)^2);
end
end
k1=(K(:, 1)+K(:,2)+K(:,3)+K(:,4));
k2=(K(:, 5)+K(:,6)+K(:,7)+K(:,8));
k3=(K(:,9)+K(:,10)+K(:,11)+K(:,12));
k4=(K(:, 13)+K(:,14)+K(:,15)+K(:,16));
p1=sqrt(k1(:,1)); p2=sqrt(k2(:,1));
p3=sqrt(k3(:,1)); p4=sqrt(k4(:,1));
P=[p1, p2, p3, p4];
n=sum(P') N=1/4*sum(P');
l=n+g.
 | | 7.2          | |
 | | -2012.3.8 | -
 | -           E | {
clc
clear
E1=[0.075,0.450,0.650,0.925]; E2=[0.150,0.250, 0.450,1.000];
E3=[0.450,0.450,0.250,0.250]; E4=[0.450,0.250,0.075,0.650];
E5=[0.450,0.650,0.250,0.075]; E6=[0.925,0.075,0.450,0.250];
E7=[0.850,0.150,0.250,0.925]; E8=[0.650,0.250,0.450,0.075];
E9=[0.250,0.075,0.250,0.000]; P=[0.075,0.925,0.850,0.150];
```

```
N=[1.000,0.150,0.250,0.925]; a=0.5;
AP1=a*[E1*P'/(norm(E1)+norm(P))]+(1-a)*[E1*P'/(norm(E1)*norm(P))];
AP2=a*[E2*P'/(norm(E2)+norm(P))]+(1-a)*[E2*P'/(norm(E2)*norm(P))];
AP3=a*[E3*P'/(norm(E3)+norm(P))]+(1-a)*[E3*P'/(norm(E3)*norm(P))];
AP4=a*[E4*P'/(norm(E4)+norm(P))]+(1-a)*[E4*P'/(norm(E4)*norm(P))];
AP5=a*[E5*P'/(norm(E5)+norm(P))]+(1-a)*[E5*P'/(norm(E5)*norm(P))];
AP6=a*[E6*P'/(norm(E6)+norm(P))]+(1-a)*[E6*P'/(norm(E6)*norm(P))];
AP7=a*[E7*P'/(norm(E7)+norm(P))]+(1-a)*[E7*P'/(norm(E7)*norm(P))];
AP8=a*[E8*P'/(norm(E8)+norm(P))]+(1-a)*[E8*P'/(norm(E8)*norm(P))];
AP9=a*[E9*P'/(norm(E9)+norm(P))]+(1-a)*[E9*P'/(norm(E9)*norm(P))];
AN1=a*[E1*N'/(norm(E1)+norm(N))]+(1-a)*[E1*N'/(norm(E1)*norm(N))];
AN2=a*[E2*N'/(norm(E2)+norm(N))]+(1-a)*[E2*N'/(norm(E2)*norm(N))];
AN3=a*[E3*N'/(norm(E3)+norm(N))]+(1-a)*[E3*N'/(norm(E3)*norm(N))];
AN4=a*[E4*N'/(norm(E4)+norm(N))]+(1-a)*[E4*N'/(norm(E4)*norm(N))];
AN5=a*[E5*N'/(norm(E5)+norm(N))]+(1-a)*[E5*N'/(norm(E5)*norm(N))];
AN6=a*[E6*N'/(norm(E6)+norm(N))]+(1-a)*[E6*N'/(norm(E6)*norm(N))];
AN7=a*[E7*N'/(norm(E7)+norm(N))]+(1-a)*[E7*N'/(norm(E7)*norm(N))];
AN8=a*[E8*N'/(norm(E8)+norm(N))]+(1-a)*[E8*N'/(norm(E8)*norm(N))];
AN9=a*[E9*N'/(norm(E9)+norm(N))]+(1-a)*[E9*N'/(norm(E9)*norm(N))];
d1=AP1/AN1; d2=AP2/AN2; d3=AP3/AN3;
d4=AP4/AN4; d5=AP5/AN5; d6=AP6/AN6;
d7=AP7/AN7; d8=AP8/AN8    d9=AP9/AN9
```

| | 7.3 | |

| | -2011.9.22 | -

| - V | {

```
clc
clear
V=[ 0.075 0.450 0.650 0.925 0.150 0.250 0.450 1.000 0.450 0.450 0.250 0.250
0.450 0.250 0.075 0.650 0.450 0.650 0.250 0.075 0.925 0.075 0.450 0.250 0.850 0.150
0.250 0.925 0.650 0.250 0.450 0.075 0.250 0.075 0.250 0.000 0.650 0.250 0.650 0.850
0.450 0.650 0.150 0.450 0.850 1.000 0.450 0.925 0.450 0.650 0.450 0.075 0.150 0.450
0.650 0.925 0.850 0.650 0.250 0.150 0.650 0.850 0.450 1.000 0.650 0.850 0.450 0.850
0.850 0.650 0.450 0.250 0.150 0.850 0.450 0.250 0.650 0.850 0.450 0.650 0.850 0.650
0.450 0.150 0.650 0.450 0.650 0.450 0.150 0.250 0.650 0.925 0.450 0.650 0.450 0.250
0.250 0.150 0.850 0.650 0.150 0.650 0.250 0.000 0.925 0.450 0.650 1.000 0.150 0.650
0.450 0.075 0.450 0.450 0.250 0.075 0.650 0.650 0.450 0.250 0.450 0.250 0.650 0.450
```

```
0.250 0.250 0.650 0.850 0.150 0.650 0.250 0.250 0.850 0.850 0.250 0.250 0.150 0.650
0.075 0.650 0.450 0.850 0.450 0.450];
    a1=V(1,:)+V(2,:)+V(3,:)+V(4,:)+V(5,:)+V(8,:)+V(9,:)+V(11,:);
    a1=a1/norm(a1);
    a2=V(6,:)+V(12,:)+V(7,:);
    a2=a2/norm(a2);
    a3=V(10,:)/norm(V(10,:)); A=8/12*a1+3/12*a2+1/12*a3;
    b1=V(13,:)+V(14,:)+V(22,:)+V(24,:)+V(23,:);
    b1=b1/norm(b1);
    b2=V(15,:)+V(16,:)+V(17,:)+V(18,:)+V(19,:)+V(20,:)+V(21,:);
    b2=b2/norm(b2);
    B=5/12*b1+7/12*b2;
    c1=V(25,:)/norm(V(25,:))
    c2=V(26,:)+V(28,:)+V(29,:)+V(30,:)+V(31,:)+V(32,:)+V(33,:)+V(35,:);
    c2=c2/norm(c2);
    c3=V(27,:)/norm(V(27,:));
    c4=V(34,:)+V(36,:);
    c4=c4/norm(c4);
    C=1/12*c1+8/12*c2+1/12*c3+2/12*c4;
    D=[A;B;C]';
    W=[0.2430,0.3178,0.0935,0.3458];
    O=W*D.
    | | 7.4             | |
    | | -2012.9.15 | -
    | -             V | {
    clc
    clear A1=[0.00 0.00 0.02 0.07 0.00 0.00 0.02 0.07 0.32 0.41 0.58 0.65 0.32 0.41
0.58 0.65 0.58 0.63 0.80 0.86 0.58 0.63 0.80 0.86 0.93 0.98 1.00 1.00 0.93 0.98 1.00
1.00 0.04 0.10 0.18 0.23 0.04 0.10 0.18 0.23 0.17 0.22 0.36 0.42 0.17 0.22 0.36 0.42
0.32 0.41 0.58 0.65 0.32 0.41 0.58 0.65 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 0.32
0.41 0.58 0.65 0.32 0.41 0.58 0.65 0.32 0.41 0.58 0.65 0.32 0.41 0.58 0.65 0.17 0.22
0.36 0.42 0.17 0.22 0.36 0.42 0.17 0.22 0.36 0.42 0.17 0.22 0.36 0.42 0.32 0.41 0.58
0.65 0.32 0.41 0.58 0.65 0.04 0.10 0.18 0.23 0.04 0.10 0.18 0.23 0.00 0.00 0.02 0.07
0.00 0.00 0.02 0.07 0.58 0.63 0.80 0.86 0.58 0.63 0.80 0.86 0.32 0.41 0.58 0.65 0.32
0.41 0.58 0.65 0.58 0.63 0.80 0.86 0.58 0.63 0.80 0.86 0.17 0.22 0.36 0.42 0.17 0.22
0.36 0.42 0.00 0.00 0.02 0.07 0.00 0.00 0.02 0.07 0.93 0.98 1.00 1.00 0.93 0.98 1.00
```

1.00 0.00 0.00 0.02 0.07 0.00 0.00 0.02 0.07 0.32 0.41 0.58 0.65 0.32 0.41 0.58 0.65 0.17 0.22 0.36 0.42 0.17 0.22 0.36 0.42 0.72 0.78 0.92 0.97 0.72 0.78 0.92 0.97 0.04 0.10 0.18 0.23 0.04 0.10 0.18 0.23 0.17 0.22 0.36 0.42 0.17 0.22 0.36 0.42 0.93 0.98 1.00 1.00 0.93 0.98 1.00 1.00 0.58 0.63 0.80 0.86 0.58 0.63 0.80 0.86 0.58 0.63 0.80 0.86 0.58 0.63 0.80 0.86 0.32 0.41 0.58 0.65 0.32 0.41 0.58 0.65 0.00 0.00 0.02 0.07 0.00 0.00 0.02 0.07 0.17 0.22 0.36 0.42 0.17 0.22 0.36 0.42 0.00 0.00 0.02 0.07 0.00 0.00 0.02 0.07 0.17 0.22 0.36 0.42 0.17 0.22 0.36 0.42 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.58 0.63 0.80 0.86 0.58 0.63 0.80 0.86 0.17 0.22 0.36 0.42 0.17 0.22 0.36 0.42 0.58 0.63 0.80 0.86 0.58 0.63 0.80 0.86 0.72 0.78 0.92 0.97 0.72 0.78 0.92 0.97];

A2=[0.32 0.41 0.58 0.65 0.32 0.41 0.58 0.65 0.58 0.63 0.80 0.86 0.58 0.63 0.80 0.86 0.32 0.41 0.58 0.65 0.32 0.41 0.58 0.65 0.00 0.00 0.02 0.07 0.00 0.00 0.02 0.07 0.04 0.10 0.18 0.23 0.04 0.10 0.18 0.23 0.32 0.41 0.58 0.65 0.32 0.41 0.58 0.65 0.58 0.63 0.80 0.86 0.58 0.63 0.80 0.86 0.93 0.98 1.00 1.00 0.93 0.98 1.00 1.00 0.72 0.78 0.92 0.97 0.72 0.78 0.92 0.97 0.58 0.63 0.80 0.86 0.58 0.63 0.80 0.86 0.17 0.22 0.36 0.42 0.17 0.22 0.36 0.42 0.04 0.10 0.18 0.23 0.04 0.10 0.18 0.23 0.58 0.63 0.80 0.86 0.58 0.63 0.80 0.86 0.72 0.78 0.92 0.97 0.72 0.78 0.92 0.97 0.32 0.41 0.58 0.65 0.32 0.41 0.58 0.65 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 0.58 0.63 0.80 0.86 0.58 0.63 0.80 0.86 0.72 0.78 0.92 0.97 0.72 0.78 0.92 0.97 0.32 0.41 0.58 0.65 0.32 0.41 0.58 0.65 0.93 0.98 1.00 1.00 0.93 0.98 1.00 1.00 0.72 0.78 0.92 0.97 0.72 0.78 0.92 0.97 0.58 0.63 0.80 0.86 0.58 0.63 0.80 0.86 0.32 0.41 0.58 0.65 0.32 0.41 0.58 0.65 0.17 0.22 0.36 0.42 0.17 0.22 0.36 0.42 0.04 0.10 0.18 0.23 0.04 0.10 0.18 0.23 0.72 0.78 0.92 0.97 0.72 0.78 0.92 0.97 0.32 0.41 0.58 0.65 0.32 0.41 0.58 0.65 0.17 0.22 0.36 0.42 0.17 0.22 0.36 0.42 0.58 0.63 0.80 0.86 0.58 0.63 0.80 0.86 0.17 0.22 0.36 0.42 0.17 0.22 0.36 0.42 0.32 0.41 0.58 0.65 0.32 0.41 0.58 0.65 0.58 0.63 0.80 0.86 0.58 0.63 0.80 0.86 0.72 0.78 0.92 0.97 0.72 0.78 0.92 0.97 0.58 0.63 0.80 0.86 0.58 0.63 0.80 0.86 0.32 0.41 0.58 0.65 0.32 0.41 0.58 0.65 0.04 0.10 0.18 0.23 0.04 0.10 0.18 0.23 0.58 0.63 0.80 0.86 0.58 0.63 0.80 0.86 0.32 0.41 0.58 0.65 0.32 0.41 0.58 0.65 0.58 0.63 0.80 0.86 0.58 0.63 0.80 0.86 0.32 0.41 0.58 0.65 0.32 0.41 0.58 0.65];

A3=[0.04 0.10 0.18 0.23 0.04 0.10 0.18 0.23 0.17 0.22 0.36 0.42 0.17 0.22 0.36 0.42 0.58 0.63 0.80 0.86 0.58 0.63 0.80 0.86 0.93 0.98 1.00 1.00 0.93 0.98 1.00 1.00 0.32 0.41 0.58 0.65 0.32 0.41 0.58 0.65 0.58 0.63 0.80 0.86 0.58 0.63 0.80 0.86 0.32 0.41 0.58 0.65 0.32 0.41 0.58 0.65 0.17 0.22 0.36 0.42 0.17 0.22 0.36 0.42 0.04 0.10 0.18 0.23 0.04 0.10 0.18 0.23 0.58 0.63 0.80 0.86 0.58 0.63 0.80 0.86 0.00 0.00 0.02 0.07 0.00 0.00 0.02 0.07 0.58 0.63 0.80 0.86 0.58 0.63 0.80 0.86 0.32 0.41 0.58 0.65 0.32 0.41 0.58 0.65 0.72 0.78 0.92 0.97 0.72 0.78 0.92 0.97 0.32 0.41 0.58 0.65 0.32 0.41 0.58 0.65 0.32 0.41 0.58 0.65 0.32 0.41 0.58 0.65 0.32 0.41 0.58 0.65 0.32 0.41 0.58 0.65 0.58 0.63 0.80 0.86 0.58 0.63 0.80 0.86 0.04 0.10 0.18 0.23 0.04 0.10 0.18 0.23 0.32 0.41 0.58 0.65 0.32 0.41 0.58 0.65 0.72 0.78 0.92 0.97 0.72 0.78 0.92 0.97 0.72 0.78 0.92 0.97 0.72 0.78 0.92 0.97 0.32 0.41 0.58 0.65 0.32 0.41 0.58 0.65 0.93

0.98 1.00 1.00 0.93 0.98 1.00 1.00 0.00 0.00 0.02 0.07 0.00 0.00 0.02 0.07 0.32 0.41 0.58 0.65 0.32 0.41 0.58 0.65 0.58 0.63 0.80 0.86 0.58 0.63 0.80 0.86 0.58 0.63 0.80 0.86 0.58 0.63 0.80 0.86 0.58 0.63 0.80 0.86 0.58 0.63 0.80 0.86 0.17 0.22 0.36 0.42 0.17 0.22 0.36 0.42 0.32 0.41 0.58 0.65 0.32 0.41 0.58 0.65 0.58 0.63 0.80 0.86 0.58 0.63 0.80 0.86 0.17 0.22 0.36 0.42 0.17 0.22 0.36 0.42 0.00 0.00 0.02 0.07 0.00 0.00 0.02 0.07 0.32 0.41 0.58 0.65 0.32 0.41 0.58 0.65 0.00 0.00 0.02 0.07 0.00 0.00 0.02 0.07 0.72 0.78 0.92 0.97 0.72 0.78 0.92 0.97 0.04 0.10 0.18 0.23 0.04 0.10 0.18 0.23 0.17 0.22 0.36 0.42 0.17 0.22 0.36 0.42 0.17 0.22 0.36 0.42 0.17 0.22 0.36 0.42];

A4=[0.17 0.22 0.36 0.42 0.17 0.22 0.36 0.42 0.04 0.10 0.18 0.23 0.04 0.10 0.18 0.23 0.72 0.78 0.92 0.97 0.72 0.78 0.92 0.97 0.58 0.63 0.80 0.86 0.58 0.63 0.80 0.86 0.04 0.10 0.18 0.23 0.04 0.10 0.18 0.23 0.58 0.63 0.80 0.86 0.58 0.63 0.80 0.86 0.17 0.22 0.36 0.42 0.17 0.22 0.36 0.42 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.93 0.98 1.00 1.00 0.93 0.98 1.00 1.00 0.32 0.41 0.58 0.65 0.32 0.41 0.58 0.65 0.58 0.63 0.80 0.86 0.58 0.63 0.80 0.86 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 0.04 0.10 0.18 0.23 0.04 0.10 0.18 0.23 0.58 0.63 0.80 0.86 0.58 0.63 0.80 0.86 0.32 0.41 0.58 0.65 0.32 0.41 0.58 0.65 0.00 0.00 0.02 0.07 0.00 0.00 0.02 0.07 0.32 0.41 0.58 0.65 0.32 0.41 0.58 0.65 0.32 0.41 0.58 0.65 0.32 0.41 0.58 0.65 0.17 0.22 0.36 0.42 0.17 0.22 0.36 0.42 0.00 0.00 0.02 0.07 0.00 0.00 0.02 0.07 0.58 0.63 0.80 0.86 0.58 0.63 0.80 0.86 0.58 0.63 0.80 0.86 0.58 0.63 0.80 0.86 0.32 0.41 0.58 0.65 0.32 0.41 0.58 0.65 0.17 0.22 0.36 0.42 0.17 0.22 0.36 0.42 0.32 0.41 0.58 0.65 0.32 0.41 0.58 0.65 0.17 0.22 0.36 0.42 0.17 0.22 0.36 0.42 0.58 0.63 0.80 0.86 0.58 0.63 0.80 0.86 0.32 0.41 0.58 0.65 0.32 0.41 0.58 0.65 0.17 0.22 0.36 0.42 0.17 0.22 0.36 0.42 0.17 0.22 0.36 0.42 0.17 0.22 0.36 0.42 0.58 0.63 0.80 0.86 0.58 0.63 0.80 0.86 0.72 0.78 0.92 0.97 0.72 0.78 0.92 0.97 0.04 0.10 0.18 0.23 0.04 0.10 0.18 0.23 0.58 0.63 0.80 0.86 0.58 0.63 0.80 0.86 0.17 0.22 0.36 0.42 0.17 0.22 0.36 0.42 0.17 0.22 0.36 0.42 0.17 0.22 0.36 0.42 0.72 0.78 0.92 0.97 0.72 0.78 0.92 0.97 0.72 0.78 0.92 0.97 0.72 0.78 0.92 0.97 0.17 0.22 0.36 0.42 0.17 0.22 0.36 0.42 0.04 0.10 0.18 0.23 0.04 0.10 0.18 0.23];

a1=sum(A1)/10;

a2=sum(A2)/10;

a3=sum(A3)/10;

a4=sum(A4)/10;

v=[0.93 0.98 1.00 1.00 0.93 0.98 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 0.93 0.98 1.00 1.00 0.93 0.98 1.00 1.00];

b1=abs(a1-v)

b2=abs(a2-v)

b3=abs(a3-v)

b4=abs(a4-v)

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| - V | {

```
clc
clear
b1=[0.5320 0.5230 0.4180 0.3640 0.5320 0.5230 0.4180 0.3640 0.7780 0.7280 0.6120 0.5540 0.7780 0.7280 0.6120 0.5540 0.7200 0.6630 0.5200 0.4580 0.7200 0.6630 0.5200 0.4580 0.4800 0.4990 0.4520 0.4190 0.4800 0.4990 0.4520 0.4190];
b2=[0.4420 0.4330 0.3100 0.2540 0.4420 0.4330 0.3100 0.2540 0.4710 0.4100 0.2520 0.1930 0.4710 0.4100 0.2520 0.1930 0.6430 0.5650 0.3980 0.3310 0.6430 0.5650 0.3980 0.3310 0.5120 0.5160 0.4520 0.4120 0.5120 0.5160 0.4520 0.4120];
b3=[0.6070 0.5960 0.4880 0.4300 0.6070 0.5960 0.4880 0.4300 0.6120 0.5600 0.4260 0.3690 0.6120 0.5600 0.4260 0.3690 0.7030 0.6370 0.4940 0.4310 0.7030 0.6370 0.4940 0.4310 0.4720 0.4690 0.3700 0.3210 0.4720 0.4690 0.3700 0.3210];
b4=[0.5970 0.5850 0.4860 0.4340 0.5970 0.5850 0.4860 0.4340 0.5940 0.5340 0.3820 0.3220 0.5940 0.5340 0.3820 0.3220 0.6220 0.5630 0.4080 0.3470 0.6220 0.5630 0.4080 0.3470 0.6300 0.6440 0.5760 0.5310 0.6300 0.6440 0.5760 0.5310];
c1=sum((b1)')/4
c2=sum((b2)')/4
c3=sum((b3)')/4
c4=sum((b4)')/4
D=[0.4593 0.4593 0.6680 0.6680 0.5903 0.5903 0.4625 0.4625 0.3598 0.3598 0.3315 0.3315 0.4842 0.4842 0.4730 0.4730 0.5303 0.5303 0.4918 0.4918 0.5662 0.5662 0.4080 0.4080 0.5255 0.5255 0.4580 0.4580 0.4850 0.4850 0.5953 0.5953];
e1=0.8*D(:,1)
e2=1.0*D(:,2)
e=1-e1-e2
```

| | 7.4 | |

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| - V | {

```
clc
clear
w=[0.32 0.41 0.58 0.65 0.32 0.41 0.58 0.65 0.72 0.78 0.92 0.97 0.72 0.78 0.92 0.97 0.17 0.22 0.36 0.42 0.17 0.22 0.36 0.42 0.93 0.98 1.00 1.00 0.93 0.98 1.00 1.00];
for i=1:4 b(i)=0.8/6*((w(i,3)-w(i,2))/(w(i,4)-w(i,1))+2);
a(i)=(b(i)*(w(i,3)+w(i,2))+(w(i,4)+w(i,1))*(0.8-b(i)))/1.6;
end
```

```
for i=1:4
d(i)=1/6*((w(i,7)-w(i,6))/(w(i,8)-w(i,5))+2);
c(i)=(d(i)*(w(i,7)+w(i,6))+(w(i,8)+w(i,5))*(1-d(i)))/2;
end
b=[0.3354 0.3413 0.3413 0.3048];
a =[ 0.4892 0.8471 0.2929 0.9745];
d=[0.4192 0.4267 0.4267 0.3810];
c =[ 0.4892 0.8471 0.2929 0.9745];
e=(a+c)/2
f=(b+d)/2
E=[ 0.5942 0.9301 0.4830 1.0331]
E=E/sum(E)
O=E*[0.1733 0.2024 0.0625 0.1675 0.3524 0.4033 0.1284 0.1486 0.0455 0.1148
0.0192 0.2656 0.0541 0.1756 0.1270 0.0715 ]
```

攻读博士学位期间主要科研成果

参与完成的项目

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2. "
" 70871121 2009-2011 ;

3. " "
12AZD109 2012-2015 ;

4. "
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